Investor Leverage Aversion and Optimal Portfolios

By Bruce I. Jacobs, PhD, and Kenneth N. Levy, CFA

Traditional portfolio theory says you should not put all your eggs in one basket. This concept, which we know as diversification, is the investor’s first defense against risk. It has been the bedrock of modern finance for decades.

But as useful and important as this concept has been, it has fallen short in safeguarding portfolios against a risk that has become increasingly common in recent years: leverage.

Modern portfolio theory, introduced by Harry Markowitz (1952, 1959), asserts that investors can reduce their exposure to the risks of individual assets by holding a diversified portfolio. Properly constructed, such a portfolio holds a combination of assets that provides the highest expected return for a given level of volatility risk, or put another way, the lowest level of volatility risk for a given expected return. Diversification generally works because the volatility risks of individual assets are partially offsetting, resulting in a portfolio that is less volatile overall than the sum of the volatilities of its constituent assets.

But when Markowitz first advanced his theory, leverage was not widely used in investment portfolios. Since then, we have witnessed the rising popularity of leveraged debt instruments and of futures and options, which facilitate leverage. We’ve also seen increased use within investment portfolios of short sales and the outright borrowing of cash.

How safe are diversified portfolios in a world where leverage is often used? Consider two portfolios having the same expected return and expected volatility. One uses leverage and the other is unleveraged. These portfolios may appear equally desirable, but they are not. The leveraged portfolio is exposed to a number of unique risks, including potential margin calls that may force liquidation of assets under adverse market conditions, the possibility of incurring losses beyond the capital invested, and the risks and costs of bankruptcy.

These events can lead to outcomes that are much worse than expected.

Diversified Portfolios

Portfolio diversification is achieved in practice through a process known as mean-variance optimization. Mean and variance are, respectively, measures of the average expected return to a portfolio and the extent to which returns differ from the average return. A utility function representing the investor’s desired trade-off between expected portfolio return and portfolio variance is applied to select the optimal portfolio, the portfolio most aligned with the investor’s preferences.

The assets selected and their weights will depend upon the expected return and the variance of return of each individual asset considered for the portfolio, as well as the covariance of returns, or the way in which a given asset’s returns are related to the returns of other assets.

To the extent that leverage increases a portfolio’s volatility, traditional mean-variance optimization recognizes some of the risk associated with leverage. But it recognizes none of the unique risks of leverage noted above. Because it assumes that investors have no aversion to the unique risks of leverage, mean-variance optimization can produce portfolios with excessive amounts of leverage.

Markowitz’s theory of diversification has succeeded admirably in convincing investors to divide their eggs among multiple baskets rather than pile them into one. However, too many investors are stacking their baskets one on top of another through their use of leverage. Eventually, the stack becomes unstable, and one misstep can lead to disaster.

Witness the saga of Long-Term Capital Management, the hedge fund that leveraged supposedly low-risk positions 25 to one in 1998; the credit crisis a decade later, brought on by leveraged securitized housing debt; the failure of highly leveraged financial institutions such as Bear Stearns and Lehman Brothers; and, in 2012, JPMorgan Chase’s multi-billion-dollar losses from its leveraged portfolio of hedges.

Incorporating Leverage Aversion

Investors need a model that allows them to explicitly account for their leverage aversion. This can be achieved by modifying portfolio theory’s mean-variance utility function, which weighs the investor’s preferred trade-off between volatility risk and return. We offer a new model in which portfolio theory’s traditional risk-aversion measure is renamed volatility aversion, and a second term representing leverage aversion is added. This gives rise to a mean-variance-leverage utility function.

The leverage-aversion term in the modified utility function assumes that the unique risks of leverage increase as a result of the interaction between a rise in the variance of the portfolio’s return and increased leverage. With this new term, investors gain the ability to trade off expected return against volatility risk and leverage risk.
Let’s examine the impact of this new specification on long-short portfolios. We’ll consider an enhanced active equity (EAE) portfolio, which maintains 100-percent exposure to an underlying stock market benchmark while allowing for short sales equal to some percentage of capital. Short-sale proceeds are then used to buy additional securities. For example, securities equal to 30 percent of capital are sold short and the proceeds are used to increase long positions by 30 percent, giving rise to an enhanced active 130-30 portfolio with leverage of 60 percent. Net exposure to the benchmark is 100 percent (130-percent long minus 30-percent short).\(^5\)

We will use the new utility function to determine the optimal amount of leverage as a percentage of capital for various levels of investor leverage aversion or, put another way, various levels of investor leverage tolerance. We plot efficient frontiers for four cases.\(^6\) In the first three, we constrain each security’s active weight to be between −10 percent and +10 percent. (“Active” weights refer to the amount by which a security’s portfolio weight exceeds or falls short of its weight in the benchmark portfolio.) The fourth frontier is computed without the constraint on active security weights.

Figure 1 illustrates how consideration of an investor’s leverage tolerance can affect the choice of optimal portfolio. The curved lines in figure 1a–c and the straight line in figure 1d represent the efficient frontier, the continuum of

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**FIGURE 1: OPTIMAL LEVERAGE FOR VARIOUS LEVERAGE-TOLERANCE (\(r_L\)) CASES**

<table>
<thead>
<tr>
<th>(r_L)</th>
<th>Expected Active Return (%)</th>
<th>Expected Active Return (%)</th>
<th>Standard Deviation of Active Return (%)</th>
<th>Standard Deviation of Active Return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100–0</td>
<td>100–0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>120–20</td>
<td>130–30</td>
<td>2.5</td>
<td>2.0</td>
</tr>
<tr>
<td>∞</td>
<td>400–300</td>
<td>2000–1900</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Source: Jacobs and Levy (2013)
optimal portfolios that offer the highest expected active return at each given level of volatility. (Volatility is measured by standard deviation, the square root of variance.) Here, “active return” refers to the amount by which the portfolio’s return is above or below the benchmark’s return.

In all cases illustrated in figure 1, the efficient frontier begins at the origin, which corresponds to the optimal portfolio when volatility tolerance is 0. In this situation the investor cannot tolerate any active volatility, so the optimal portfolio is an index fund. By definition, an index fund provides zero expected active return and zero standard deviation of active return.

In the first case (figure 1a), leverage tolerance is 0 (τL = 0), meaning the investor is unwilling to use leverage and holds a long-only portfolio. As the investor’s tolerance for volatility increases, the optimal portfolio moves out along the frontier, taking on higher levels of standard deviation to earn higher levels of return. These portfolios take more-concentrated positions in higher-expected-return securities as volatility tolerance increases. As noted, every portfolio along the frontier is a “100-0” portfolio, meaning it is invested 100-percent long, with no short positions.

The second case (figure 1b) illustrates the efficient frontier where the investor has a moderate tolerance for leverage (τL = 1). As tolerance for volatility increases, the optimal portfolio moves out along the efficient frontier, achieving higher levels of expected return with higher levels of volatility, as in the first case. With moderate leverage tolerance, however, the investor can achieve a higher return at any given risk level than the investor with zero leverage tolerance.

As the plot indicates, increasing volatility is accompanied by increasing leverage. The optimal portfolio ranges from a 100-0 long-only portfolio to a 130-30 enhanced active portfolio. For the investor with moderate leverage tolerance, any of these portfolios can be optimal, depending on the level of volatility tolerance. Higher risk-return portfolios can be achieved with less concentration of positions when leverage is allowed than when leverage is not allowed.

The third case (figure 1c) illustrates the efficient frontier for an investor with infinite leverage tolerance (τL = ∞). As discussed earlier, traditional mean-variance optimization implicitly assumes investors have infinite tolerance for the unique risks of leverage; thus mean-variance optimization provides the same result as mean-variance-leverage optimization when the investor has infinite leverage tolerance. As the investor’s volatility tolerance increases, the optimal portfolio goes from zero leverage to enhanced active portfolios of 200-100 to 400-300. The investor with infinite leverage tolerance is not concerned about the unique risks of leverage. This investor can achieve a higher expected return at any given level of standard deviation of return, albeit with increasing leverage risk.

The last case (figure 1d) is identical to the third, except the 10-percent constraint on individual security active weights is removed. No leverage aversion is assumed, and no constraints are placed on individual position sizes. The optimal portfolios all hold the same proportionate active security weights but apply increasing levels of leverage as volatility tolerance increases. Because each portfolio is just a leveraged version of the same set of active positions, the efficient frontier is simply a straight line. In this case, ever-higher levels of leverage are used to achieve ever-higher expected returns along with ever-higher standard deviations of return.

Efficient Frontiers for Various Leverage-Tolerance Levels

Figure 2 displays five efficient frontiers on one chart. Each frontier corresponds to a different level of leverage tolerance. Here, zero leverage tolerance again represents an investor unwilling to use
We propose that investors determine optimal levels of leverage based on their degrees of leverage aversion.

Investment, and higher efficient frontiers correspond to investors with greater tolerances for leverage. The 10-percent active security weight constraint applies to all the portfolios.

It might at first appear that the highest level of leverage tolerance results in the dominant efficient frontier; that is, higher leverage allows the investor to achieve higher returns at any given level of volatility. But when leverage aversion is considered, it becomes apparent that each frontier consists of the set of optimal portfolios for an investor with the given level of leverage tolerance.

For example, consider the three portfolios represented by the points labeled A, B, and C in figure 2.

(Tables and figures are omitted for brevity.)
lions will better reflect their preferences. A lower level of leverage in the market also may reduce the systemic risk that has repeatedly roiled the global financial system.

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Endnotes

1 Certain legal entities, such as limited partnerships and corporations, can limit investors’ losses to their capital in the entity. Losses in excess of capital would be borne by others, such as general partners who have unlimited liability or prime brokers.

2 Investors who use leverage usually limit it, choosing a leverage level and imposing it on the portfolio with a constraint. Markowitz (1959) showed how to implement constraints. We propose in this article a method of determining the optimal amount of leverage based on an investor’s aversion to leverage risk.

3 See Jacobs and Levy (2012).

4 For details, see Jacobs and Levy (2013).

5 For purposes of this discussion, we assume the strategy is self-financing and there are no additional costs. In practice, there would be financing costs (such as stock loan fees); furthermore, hard-to-borrow stocks may entail higher fees. For more on EAE portfolios, see Jacobs and Levy (2007).

6 The frontiers are computed subject to standard constraints, including one that requires the portfolio to be fully invested, and another that requires the portfolio’s beta (the degree to which its movements correspond to the movements of the market portfolio) to equal the benchmark’s beta.

7 As with figure 1c, the same efficient frontier is derived whether the investor uses conventional mean-variance optimization or mean-variance-leverage optimization, since the investor is assumed to have no aversion to leverage.

8 Expected active returns shown do not reflect any costs associated with leverage-related events, such as forced liquidations at adverse prices or bankruptcy. These costs, however, are reflected in the disutility implied by the leverage-aversion term.

References


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