The Relative Asset Pricing Model: Toward a Unified Theory of Asset Pricing

By Arun Muralidhar, PhD, Kazuhiko Ohashi, PhD, and Sunghwan Shin, PhD
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Abstract

Since the development of modern portfolio theory (MPT) in the late 1950s and early 1960s, academics have offered numerous competing theories. MPT’s simplicity is appealing: The expected return on an asset is simply a function of the return of the market portfolio and the asset’s beta to the market portfolio. Arbitrage pricing theory (APT) suggests that factors over and above the market portfolio may be relevant; empirical tests somewhat validate APT, though APT does not address specific factors. Prospect theory or behavioral finance (PT) finds that investors do not behave as predicted by MPT, and that MPT seems to lack a reference point. The adaptive markets hypothesis (AMH) lacks a quantitative theory but suggests that applying principles of evolution, competition, adaptation, and natural selection to financial interactions may explain some observed market phenomena. This paper boldly claims that it is possible to capture many aspects of MPT, APT, PT, and AMH in a simple paradigm, the relative asset pricing model (RAPM), which is anchored in how investors behave and act. In RAPM, at the highest level of application, assets are invested to maximize surplus or funded status, not to maximize wealth, thereby providing a reference point—liabilities—as desired by PT. At a secondary level, asset management is delegated to agents and they make decisions relative to subcomponents of the liability proxy, with specific constraints because delegation leads to certain challenges. Depending on how liabilities have been proxied, RAPM credibly rationalizes many additional factors postulated in APT. RAPM also can absorb AMH because as regulations have changed and investors have become more sophisticated and creative in delegation, investors have adapted how they have proxied liabilities/structured portfolios. If the simple RAPM postulated synthesizes all these attributes, it could provide the basis for a unifying theory of asset pricing, demonstrating that all current branches of asset pricing are merely different perspectives of a multi-dimensional RAPM, and additional research will flesh out RAPM in more detail and greater complexity.

Some people believe that it is excellent and correct to work out a thing as absurd as did that Sarmatian astronomer who moves the earth and stops the sun. Indeed, wise rulers should have curbed such light-mindedness. —Philipp Melanchthon

Introduction

The many competing and credible theories of asset pricing and allocation include modern portfolio theory (MPT), arbitrage pricing theory (APT), and prospect theory/behavioral finance (PT), and investors face the conundrum of which to follow. This paper suggests that these various theories could fit comfortably under a simple approach to financial markets that we call the relative asset pricing model (RAPM). RAPM requires a change in perspective plus a reference point that anchors asset pricing/allocation theory. RAPM calls for theorists and practitioners to view asset pricing from a new and novel perspective: the specific behavior of investors, who are driven by the desire to satisfy liabilities, so this is the true reference point for all decisions and should be the driver of asset prices. This could be controversial for many given the current focus of asset pricing theory. We dare to say that this new perspective is parallel to Nicolaus Copernicus in the 16th century challenging conventional wisdom by positing that the earth moves around the sun, which in the day proved difficult for Protestant theologian Philipp Melanchthon and others in the ruling class (Milosz 1983, 38).

Some have attempted to integrate these theories. Merton (1973) has been used to suggest integrating MPT and APT, and others have used agency theory (Cornell and Roll 2005; Brennan and Li 2008; Basak and Pavlova 2012) or peer comparison in an MPT framework (Reisman and Lauterbach 2002) to suggest the basis for APT factors; Reisman and Lauterbach (2002) use their model to demonstrate why investors may have a domestic bias that could cause them to deviate from optimal portfolios of MPT. On the other hand, Barberis et al. (2001) establish asset prices using prospect theory assumptions embedded in a utility theory approach. Das et al. (2010) integrate appealing features of Markowitz’s mean-variance portfolio theory and behavioral portfolio theory into a mental accounting framework. This paper shows how aspects of all these theories can be captured in a common framework, with a relatively simple and practical twist to MPT.

MPT, which comprises Harry Markowitz’s mean-variance optimization (MVO) and the Sharpe-Linter-Tobin-Mossin capital asset pricing model (CAPM), is the backbone of modern finance; see Lintner (1965), Markowitz (1952a), Mossin (1966), Sharpe (1964), and Tobin (1958). Retail and
institutional investors globally use MPT to structure portfolios because it is simple and elegant. MPT assumes that investors derive utility from wealth and are averse to volatility, and it utilizes a relatively simple equilibrium model that provides robust recommendations for asset pricing and asset allocation. For example, to establish the expected return on an asset, only the expected return on a market portfolio, the risk-free rate, and the asset's beta are required. To paraphrase Markowitz (2005, 18), in its purest interpretation, all investors, whether retired widower or young investor, will allocate their assets between the risk-free asset and the market portfolio, with differences between individuals or institutions differing only in their desire for/aversion to volatility (called two-fund separation). In practice, most investors do not appear to implement two-fund separation but they do use CAPM inputs on returns to structure portfolios using MVO. Further, many of them use MPT and have an objective function different from that postulated in the theory. Finally, for practitioners, MPT provides an appealing framework to evaluate risk-adjusted performance and this aspect of the theory is vital (though often overlooked) to measuring the quality of investment opportunities and investment decisions.

Within ten years of publication of these seminal CAPM papers, two different approaches questioned MPT's validity, because empirical research did not appear to bear out the assumptions or the conclusions. The first, PT, went to the heart of MPT and questioned whether investors truly behaved as hypothesized and whether they derived utility from wealth (Kahneman and Tversky 1979). Instead, tests conducted across varied groups of individuals revealed that individuals were more concerned about gains and losses and appeared to demonstrate loss aversion, which did not seem consistent with MPT behavior. For example, an MPT investor gets the same utility from $1 million in wealth, regardless of whether it was achieved by losing $500,000 from initial wealth of $1.5 million or by gaining $500,000 from an initial wealth of $500,000. Kahneman (2011) succinctly summarizes MPT's primary shortcoming: "[T]he previous theory fails because it fails to talk about reference points." In its initial form, however, PT was unable to offer an equally robust theory on how to forecast expected returns or structure portfolios, or calculate risk-adjusted performance.

The second group that diverged from MPT felt the model was too limited in postulating that all returns could be estimated with just a single market factor and a beta parameter. APT is a general theory of asset pricing in that the expected return of a financial asset can be modeled as a linear combination of factor returns of various macro and economic factors including the market factor, and a risk-free rate. In APT, the sensitivity to each factor return is represented by a factor-specific beta coefficient (Ross 1976). In the case of the one-factor APT where the factor is a market factor, APT exactly coincides with CAPM. Under this approach, in addition to a proxy for the market factor, empirical tests conclusively show that factors such as economic growth, inflation, slope of the yield curve, term premiums, market capitalization (or size), and type of asset (value or growth) matter. The challenge with APT is that the theory does not specify clear factors, so their determination could be attributed to data mining. This flexibility, however, allows extended MPT models, à la Merton (1973), Cornell and Roll (2005), Brennan and Li (2008), and even Reisman and Lauterbach (2002), to offer a path to integrate MPT with APT. More important, APT was unable to offer insight into portfolio construction or risk-adjusted performance, because again it was not derived from a model based on the behavior of agents, but rather the exclusion of arbitrage opportunities.

The adaptive markets hypothesis (AMH), though qualitative in nature and focused on providing an alternative to the efficient markets hypothesis (EMH), attempts to explain certain phenomena with the fact that markets and opportunities are dynamic and market participants adapt to circumstances. AMH, though not fully developed, applies the principles of evolution, competition, adaptation, and natural selection to financial interactions and may explain some observed phenomena. As Lo (2004) notes:

[M]uch of what behavioralists cite as counterexamples to economic rationality, loss aversion, overconfidence, overreaction, mental accounting, and other behavioral biases are, in fact, consistent with an evolutionary model of individuals adapting to a changing environment via simple heuristics. … Individuals make choices based on past experience and their ‘best guess’ as to what might be optimal, and they learn by receiving positive or negative reinforcement from the outcomes.

Again, because this approach lacks a quantitative model, it is hard to make expected return forecasts or get concrete proposals for asset allocation.

One challenge to testing MPT’s validity is that it treats both bonds and stocks as “risky assets” even though they are largely different instruments. In simplest terms, bonds provide predictable cash flows and stocks do not, and this difference changes their value to investors. In addition, MPT has largely been tested on stocks, even though MPT assumes the market portfolio includes all assets. The danger of this exclusive focus on stocks (or treating stocks the same as bonds) is that it risks ignoring the unique characteristics of a substantial part of capital markets. Indeed, equities have declined from more than 50 percent to about 35 percent of the estimated global market (Doeswijk et al. 2012), and bonds increasingly have become the dominant asset in institutional portfolios.

So how does one integrate all these findings that are at odds with MPT? Copernicus’ seventh assumption provides an interesting clue:

[T]he apparent retrograde and direct motion of the planets arises not from their motion but from the
A truly unifying theory of these various branches of financial theory would need to synthesize seemingly disparate views into a seamless internally consistent theory that ideally also incorporates bonds in a robust manner. To this end, the goal of this paper is to suggest a change of the reference point for asset pricing theory and propose a model that has the potential to resolve some of the seemingly retrograde and unequal aspects of asset pricing.

In effect, a suitable theory needs to capture the best aspect of each of the disparate theories and clearly recognize actual investor behavior. At a minimum, it must incorporate the following characteristics:

- **From modern portfolio theory (MPT)**: The attractive equilibrium and utility-based model approach that includes clear prescriptions for potential returns, reasonable asset allocation, and measures of risk-adjusted performance
- **From arbitrage pricing theory (APT)**: An ex-ante rationale for additional factors and a possible explanation for the seeming importance of certain factors
- **From prospect theory (PT)**: A credible reference point and the potential for explaining preferences based on gains/losses and loss aversion
- **From the adaptive markets hypothesis (AMH)**: How investors have adapted to changing markets, regulations, and opportunities

### Key Aspects of Existing Theories

#### Modern Portfolio Theory

Modern portfolio theory comprises the capital asset pricing model (CAPM) and mean-variance optimization (MVO). For a detailed overview of MPT, including defenses against various critiques, see Levy (2011).

CAPM suggests that if investors had mean-variance preferences, as assumed by Markowitz (1952a), then the expected returns of any asset, defined as \( r(i) \), can be explained by a simple formula such as equation 1; its excess over the risk-free rate \( r(F) \) is equal to beta times the excess of the return of the market portfolio \( r(M) \) over the risk-free rate. Beta is defined as the covariance of the asset \( r(i) \) to the market portfolio \( r(M) \) divided by the variance of the market portfolio \( r(M) \), as shown in equation 2.

\[
E[r(i) − r(F)] = \beta × [E[r(M) − r(F)]] \tag{1}
\]

\[
\beta = \frac{\text{cov} [r(M), r(i)]}{\text{var}[r(M)]]} \tag{2}
\]

The fundamental assumption here is that investors derive utility from wealth and are averse to volatility. Friedman and Savage (1948) and Markowitz (1952b) questioned this assumption because it makes it hard to reconcile the fact that individuals appear to both gamble and seek insurance.

Markowitz (1952b) solved this quandary by focusing on utility changes in wealth. This is effectively where PT begins. Utility from changes in wealth, however, was not used in MPT.

The simplicity of this formula led to its widespread use. To maximize the utility of wealth given an opportunity set of assets, all investors would split their portfolios between the market portfolio and the risk-free asset based on their risk aversion. The riskless position invests the entire portfolio in an asset with zero volatility and zero correlation with other assets, i.e., entirely in \( r(F) \). Because no such asset truly exists and all assets have non-zero volatility, Black (1972) recommended the use of a zero-beta CAPM, where the zero-beta asset would have volatility and a return, but with zero correlation to the market portfolio. The resulting formulae looked very similar to equation 1, with the risk-free asset replaced by the zero-beta asset, and the beta formulation modified accordingly. Black (1972), however, did not specify this zero-beta asset.

A comprehensive, user-friendly model for investors should include recommendations for asset allocation, models for asset pricing, and methods for calculating risk-adjusted performance—the three key aspects of effective asset management. The last aspect, calculating risk-adjusted performance of investment managers or investment opportunities, is critical because it helps to evaluate investment opportunities and structure portfolios, as illustrated in figure 1.

MPT is one of the few theories that easily lends itself to robust models for each of the three key aspects of investment management shown in figure 1. Treynor (1966), Sharpe (1994), and Modigliani and Modigliani (1997) derived risk-adjusted performance measures from the MPT equilibrium model to evaluate the efficacy of a particular investment or investment manager to whom management of assets may be delegated. The two most commonly used risk-adjusted measures are the Sharpe ratio (equation 3) and the differential Sharpe or information ratio (equation 4). The Sharpe ratio
measures risk-adjusted performance of an asset or manager in isolation (or relative to the risk-free asset), whereas the information ratio captures the risk-adjusted performance of an investment manager relative to some market benchmark. One can see how these measures are linked to equation 1.\textsuperscript{4} Moreover, the Sharpe ratio can be seen as a specific case of the more general information ratio, where the benchmark is the theoretical risk-free asset without volatility and zero correlation to other investments.\textsuperscript{5} We discuss how the use of these measures may have led to the evolution of portfolio management, particularly AMH, in more detail below.

Sharpe ratio \( (i) = \frac{E[r(i)] - r(f)}{\sigma(i)} \) \hspace{1cm} (3)

Information ratio \( (i) = \frac{E[r(i) - r(B)]}{\sigma(i) - 2 \times \rho(i,B) \times \sigma(i) \times \sigma(B) + \sigma^2(B)} \) \hspace{1cm} (4)

where \( r(B) \) is the return of the benchmark portfolio and \( \sigma^2(j) \) is the variance of asset \( j \) and \( \rho(i,B) \) is the correlation between asset \( i \) and the benchmark.

Muralidhar (2000) demonstrated how the Sharpe ratio, Modigliani and Modigliani (1997), and the information ratio do not rank external managers based on skill. The argument is that, in relative comparisons, where the volatility of excess returns (relative to the benchmark) is the measure of risk, multiple moving parts (volatilities and correlations) need to be normalized. Hence, commonly used risk-adjusted performance measures may be inappropriate for relative comparisons. We discuss this further in the “Implications” section.

Prospect Theory

PT evolved from researchers asking diverse groups of individuals in laboratory settings specific questions about their attitude to contrived gambles to examine whether they truly derived utility from wealth and whether they displayed the requisite risk aversion. Levy (2011) summarizes the PT argument: “Tversky and Kahneman find that investors maximize expected value of some preference that is neither concave nor convex; investors make investment decisions based on the change of wealth rather than the total wealth, contradicting Expected Utility Theory.” PT uncovered that, in certain portions of the wealth spectrum, rather than being risk averse investors actually sought risk. Loss aversion—the desire to avoid sure losses more than potential gains—seemed widespread. In effect, rather than a typical utility function, individuals appear to have a value function similar to that shown in figure 2. The key point of the PT critique, as articulated in Kahneman (2011), is that “the previous theory fails because it fails to talk about reference points.” In other words, utility did not appear to be derived from absolute levels of wealth.

PT has added more complex models to explain investor behavior, including models to capture overconfidence and market overreaction to news. Evidence indicates that individuals create mental accounts for various aspects of their lives (e.g., savings for retirement, children’s college fund, and spending on luxury goods for retail investors; pension fund portfolios, retiree health portfolios, and endowment portfolios for institutional investors) as opposed to thinking about all their monies as a single portfolio. Each subportfolio is associated with a goal, and each goal has a return threshold that investors would not tolerate dipping (with some probability) below. Das et al. (2010) suggest that investors consider efficient frontiers for each mental account, and make decisions for that subportfolio subject to achieving the threshold return in an MVO setting. In effect, the risk-aversion parameter in the MPT setting is implied from this threshold approach, and the reference point sought by PT is the threshold return. This approach captures the concept of mental accounts and attempts to integrate MPT with PT. This approach to explaining asset allocation in a mental accounts/PT framework is not extended into an equilibrium-return forecast model, and one assumes that the appropriate risk-adjusted performance measure would be the information ratio for each mental account (because each account would have a different benchmark).

Barberis et al. (2001) attempt to capture prospect theory in a utility-theory framework. In addition to the utility of consumption, they first add a utility of changes in financial wealth to incorporate aspects of prospect theory. The utility of changes in financial wealth is based on the performance of risky assets and on the assumption of loss aversion. More importantly, this loss aversion is based on some reference benchmark level of stocks (e.g., some historical price) and that becomes the state/exogenous variable and the reference point.\textsuperscript{6}

The Barberis et al. (2001) model has many attractive aspects. First, it captures the PT finding that investors worry about changes in wealth, not wealth per se. Second, loss aversion is explicit in the utility function, reflecting the investor’s greater sensitivity to wealth reduction than to equivalent gains. Third, risk aversion changes with time, because investors become more risk averse when the stock market declines.

**Figure 2: Preferences Under Prospect Theory**

![Figure 2](source: Kahneman and Tversky 1979, figure 3)
(i.e., prior investment declines) and vice versa. Fourth, under some specific conditions, they are able to recover the PT value function. Finally, they are able to extract asset prices in this complex setting.

Prospect theory, however, has not caught on or displaced MPT. Barberis (2013) highlights the challenge emanating from prospect theory:

[T]he reason that developing applications of prospect theory in economics is taking a long time is because it is not always obvious how, exactly, to apply it. The central idea in prospect theory is that people derive utility from ‘gains’ and ‘losses’ measured relative to a reference point. But in any given context, it is often unclear how to define precisely what a gain or loss is, not least because Kahneman and Tversky offered relatively little guidance on how the reference point is determined. ... Are they gains and losses in overall wealth, in the value of total stock market holdings, or in the value of specific stocks?

In the section below titled “The Case for Relative Asset Pricing: Liabilities and Agency,” figure 3 from IMF (2013) appears to be at odds with this assumed relationship between stock valuations and risk aversion. Therefore, in the section on RAPM below, we develop another approach to the concept of a threshold return for each mental account and still try to develop a model to forecast expected returns and capture more complex risk preferences. In addition, we effectively anchor the relative decision making to a practical and tangible benchmark that all investors experience as opposed to some arbitrary price level for either the entire stock market or a particular stock, or even some notion of absolute wealth because this paper focuses on relevant and practical measures of relative wealth.

**Arbitrage Pricing Theory and Fama-French**

Ross (1976) assumes neither risk aversion nor normality of returns, but rather looks to explain the returns of an asset by a series of economic variables. A stock’s price depends on the expected cash flows it will provide and also on expected returns that discount expected cash flows back to the present. Chen et al. (1986) summarize the case for the APT: “Although stock market indices ‘explain’ much of the intertemporal movements in other stock portfolios, their estimated exposures (their betas) do not explain cross-sectional differences in average returns after the betas of the economic state variables have been introduced.” Instead, Chen et al. (1986) focus strictly on stocks and suggest that, at a minimum, industrial production, inflation (expected and unexpected), the term structure of yields, and a risk premium (defined as the spread between BAA and government bond yields) should be important.

APT states that, if asset returns follow a factor structure, the relationship in equation 5 exists between expected returns and factor sensitivities:

\[
E[r(i)] - r(F) = \beta_{i1} \times RP_1 + \beta_{i2} \times RP_2 + ... + \beta_{in} \times RP_n
\]

where \(RP_k\) is the risk premium of the \(k\)th factor, and \(\beta_{ik}\) is the factor sensitivity of the \(k\)th factor for the \(i\)th asset.

Fama and French (1992, 2004) critique CAPM and state that “the empirical record of the model is poor—poor enough to invalidate the way it is used in applications.” They suggest instead a three-factor model based on empirical tests, and the three factors are (1) the market; (2) the small-cap effect; and (3) the value effect (Fama and French 2004). Later researchers such as Carhart (1997) added a fourth factor: momentum. Duarte (2013) suggests an inflation effect (in addition to factors identified in Fama and French 2004, hereafter called Fama-French factors) in that “investors are willing to accept lower unconditional returns when holding securities that are good hedges against inflation.” As Fama and French (2004) note:

[F]rom a theoretical perspective, the main shortcoming of the three-factor model is its empirical motivation. The small-minus-big (SMB) and high-minus-low (HML) explanatory returns are not motivated by predictions about state variables of concern to investors. Instead they are brute force constructs meant to capture the patterns uncovered by previous work on how average stock returns vary with size and the book-to-market equity ratio.

We discuss below why these factors are important from a RAPM perspective, how they fit with observed investor behavior, and why when expressed as spreads (e.g., small minus large; BAA minus government yield, value minus growth), they potentially lay the groundwork to validate RAPM.

**Adaptive Markets Hypothesis**

Finally, because markets are dynamic and investors are not necessarily irrational, a new branch of economics has attempted to apply concepts of evolutionary biology to the evolution of markets and market participants. The adaptive markets hypothesis (AMH), which focuses on survival of the fittest and the notion that investors and agents adapt to changing regimes, is still under development. Lo (2004) notes its implications:

[T]he first implication is that to the extent that a relation between risk and reward exists, it is unlikely to be stable over time. Such a relation is determined by the relative sizes and preferences of various populations in the market ecology, as well as institutional aspects such as the regulatory environment and tax laws. As these factors shift over time, any risk/reward relation is likely to be affected. A corollary of this implication is that the equity risk premium is also time-varying and path-dependent. ... [Second,] from an evolutionary perspective, the existence of active liquid financial markets implies that profit opportunities must be present. As they are exploited, they disappear.
A third implication is that investment strategies will also wax and wane, performing well in certain environments and poorly in other environments.

The Case for Relative Asset Pricing: Liabilities and Agency

How Institutional Investors Behave

For simplicity, we focus on pension funds as the representative institutional investor. This investor-type is global, highly formalized, well-researched, and has been growing in proportion to other investors. As the IMF (2011) survey highlights, the role of institutional asset management has grown dramatically and pension funds form a significant component of the total. The concepts applied to this institutional investor apply to retail investors, insurance funds, endowments, sovereign wealth funds, and even foundations. The goal of this section is to provide practical insight into the market ecology, highlight how these investors make decisions, and then attempt to fit this investment behavior into an investment paradigm/model, reconciling the various theories to this broader model.

A pension fund’s raison d’être is to accumulate sufficient assets, along with ongoing contributions, to pay the pension promises (liability) of the sponsoring entity (principal). The parallel raison d’être for individuals (retail investors) is to save/accumulate assets and is described as the life cycle hypothesis (Modigliani and Ando 1963). Institutional liabilities usually are expressed as a series of cash flows into the future, which creates a conundrum for the investment team because cash flow is not the language they speak. They typically think in terms of asset classes, securities, returns, valuation of assets, and volatility. As a result, the first step in the investment process is a sort of translation, where the investment team needs to develop a “proxy liability,” expressed in asset-speak, mimicking or closely tracking future cash outflows, and allowing for a frequent valuation of the present value of all cash flows—a process called marking-to-market. Typically principals do not have the expertise to manage assets and they hire agents to make the actual decisions to implement portfolios. Figure 3 shows a simple two-step process that most investors go through, intentionally or unintentionally, to manage their assets relative to this target liability.

Step 1 of this exercise is for the principal to set a strategic asset allocation (SAA) meant to proxy the liability portfolio and capture all the fund’s objectives. A typical SAA is composed of a series of benchmark indexes (e.g., S&P 500, MSCI EAFE), each with a target weight such that the sum of the weights totals 100 percent. In effect, this step is an attempt to convert the stream of projected liability cash outflows that need to be serviced into a portfolio of securities/assets, projected to achieve the objective of generating sufficient value to defease payments, within stated risk parameters. This attempt to translate/convert the cash flows into a formal asset construct gives us a proxy liability. A strong case has been made that when cash flows can be projected with some certainty, it is easy to create a bond portfolio that closely tracks cash flows (Leibowitz 1986). Though one can argue that our representative investor, the pension fund, has some of the most predictable of investor cash flows to service, historically the SAAs of global pension funds and other institutional investors have included many non-bond assets with substantial allocations.

In the early 1990s, U.S. pension funds commonly had a simple SAA of 60-percent S&P 500 Equity Index/40-percent Lehman Aggregate Bond Index of four- to six-year duration. This SAA seems at odds with the standard cash-flow matching concepts one would use to proxy liabilities today. Therefore, one could question the equity allocations or even the target duration in the SAA, because a typical mature pension fund has a duration greater than ten years. It can be argued that, in the 1980s and 1990s, the SAA was not intentionally designed to track cash flows, because regulations did not require it, and MPT, the entrenched model used to develop the SAA, largely ignored it; Sharpe and Tint (1990) address the importance of liabilities, especially surplus (or assets minus liabilities) from an asset allocation perspective, but sadly this paper is not considered part of MPT. As a result, the SAA became the de facto liability proxy from a theoretical and a practical point of view. This is because the SAA, even if it tracks liabilities very poorly from a cash-flow matching perspective, once articulated and approved by the principal, becomes the benchmark for measuring investment performance and determining job security and compensation. In effect, this SAA drives all investment decisions, which impacts markets, asset pricing, and asset performance.³

Markets are dynamic and a portfolio can never be maintained at the fixed SAA weights, so the vast majority of portfolios have two major alternatives for a clear rebalancing policy: The portfolio typically is rebalanced back to the SAA weights at the end of some calendar period (e.g., quarter) or ranges are specified around the SAA with the restriction that the portfolio is rebalanced back to the SAA (or some point within the range) when a limit is breached.

More recently, some funds have created separate liability proxies and SAAs. Sophisticated corporate pension funds and Dutch and Danish funds proxy their projected cash outflows with a replicating bond or swap portfolio that is then the liability proxy. They then select an SAA that is different from the liability proxy to guide investment decisions (Muralidhar and van Stuijvenberg 2005). In some cases, e.g., the Netherlands, the regulator is very particular about the method and interest rates used to measure how the liability has performed to help
Investing has evolved in the past five decades along two fronts—first, from a largely asset-only approach to a more asset-liability approach, also known as liability-driven investing or LDI; and second, from relatively simple portfolios to extremely complex portfolios (IMF 2013). We briefly review these trends because they impact various theories and also provide context for RAPM.

This evolution is the crux of AMH. One trend that tops these two trends, however, is the greater involvement of institutional investors in financial markets, as highlighted in IMF (2011), because this trend validates our choice in RAPM of a pension fund/institutional investor as the representative investor. Using AMH terminology, investment ecology and ecosystems have changed dramatically. Institutional investors are much more likely to be relative asset pricing investors because institutional funds tend to exist to serve a specific purpose—pensions, insurance claims, or university or sovereign spending—whereas retail mental accounts can be more amorphous. One can argue that many mental accounts in retail portfolios also will be managed relative to some clearly projected expenditure/liability, so they also are RAPM-based. The case can be made, however, for some subportfolios to be based on the absolute pricing process of MPT and seeking wealth for wealth’s sake, as opposed to the relative wealth approach under RAPM.

MPT was widely adopted for its simplicity and the fact it was taught extensively in business schools and certification programs. One can see traditional MPT as an absolute asset-pricing model because decisions are made relative to a risk-free numeraire and are asset-only in approach. Though Sharpe and Tint (1990) makes the case for investing assets relative to liabilities and maximizing surplus, through the 2000s most U.S. funds adopted a largely asset-only, MVO approach. Even in the Netherlands, where LDI drew attention as early as the 1990s, most portfolios were structured with substantial equity components, even though liabilities were bond-like and many were more than fully funded. The equity market crash in 2000–2002 and the related rally in bonds led to a substantial decline in funding, and many funds fell below full funding. This, in turn, led to a change in regulation in the Netherlands, the United States, the United Kingdom, and other countries, requiring these institutions to be fully funded and with a greater emphasis on marking liabilities to market using long-duration interest rates. As a result, U.S. corporate pension funds and all Dutch, Danish, and Finnish funds began to adopt a much more stringent LDI approach with a greater emphasis on long-duration bonds as the liability proxy and an allocation to risky assets to try to overcome the underfunding; see Amenc et al. (2010) for a model on this in continuous time. One could argue the further decline in long-term rates in the 2000s was driven by demand for longer-term bonds triggered by stricter regulation (which in turn hurts corporate funds as the liabilities get marked higher).

Public pension plans in the United States, which are governed by Government Accounting Standards Board (GASB) rules, use a much higher discount rate than corporate plans, which provides a disincentive to proxy liabilities with...
long-duration bonds and instead do so with equity portfolios. This is a source of much debate (Rauh and Novy-Marx 2011). Figure 4 presents an analysis of risk aversion of a select group of institutional investors based on their funded status during 2001–2011. This data from IMF (2013), based on allocation information for the least-funded 10 percent of U.S. public pension plans, demonstrates a few noticeable characteristics. First, as the funded status of public pension plans began to decline from 2001 onward, the allocation to risky assets increased. In other words, risk aversion declined with funded status, leading to a marked increase in alternative assets as represented by the size of the bubbles. This is consistent with a variation of PT; figure 4 demonstrates loss aversion, but based on funded status as opposed to wealth. Second, clearly risk aversion is also dynamic in time, as suggested by AMH. Third, 2003–2007 had a generally upward trend in equity valuations, which potentially contradicts an assumption made in the utility function in Barberis et al. (2001). Based on this model, one would assume that as equity prices rose, risk aversion also should have risen, but in this subset of funds risk aversion actually declined substantially. This opens the door to an alternative approach that links risk aversion to funded status rather than equity levels. This behavior is also validated in a more recent study of 126 public pension plans.12

Similarly, even corporate funds lack consensus about whether the liability is the economic liability (cash flows discounted at the appropriate risk class rate and marked-to-market) or the funding liability (i.e., the discount rate authorized by the regulator which is a modified marked-to-market). Most funds go with the rate permitted by the regulator or the appropriate accounting standard boards, not the economic liability. To summarize, funds have moved from absolute pricing to relative pricing, and the benchmark to which relative decisions are evaluated, within certain industry segments, has evolved with changing regulation. Further, the link between risk aversion and funded status is closer than the link between risk aversion and equity values or wealth.

In terms of the assets that these funds invested in, the markets have evolved from portfolios that were largely invested in bonds in the 1950s and 1960s, to a post-MPT portfolio 40 percent in bonds (short duration) (large cap, growth, and domestic) and 60 percent in equities (private equity, small caps, real estate, hedge funds). Figure 4 shows the increased allocation to risky assets and a growing use of alternative investments not highly correlated to liabilities. During the 2008 crash many of these strategies underperformed while liabilities rose in value as interest rates collapsed.13 In the United States, there were concerted efforts to provide funding relief for pension funds to allow them to smooth the discount rate (e.g., MAP 21, which allows plan sponsors to measure pension liability using the twenty-five-year average of segment rates plus or minus a corridor) and this changed the demand for bonds. Further, a belief crept into the markets that diversification had failed. Most funds now seem to be taking a risk-parity approach in an attempt to correct for perceived overreliance on equity risk in typical portfolios (corrected by increasing allocations to longer-duration fixed income). Finally, in the United States, the rating agency Moody’s has decided to apply a market-based rate to discount public pension plan liabilities, which would be significantly lower than the current expected returns used, and this also may change the demand for certain assets in funds influenced by Moody’s ratings of county and state debt. These casual observations from a practitioner’s point of view seem to agree with the limited research that’s been done on the evolution of the global market portfolio, such as Doeswijk et al. (2012), which provide an analysis of market trends across multiple asset classes during 1990–2011. Doeswijk et al. (2012) examined the global market portfolio from the...
In the interest of simplicity, we consider Step 1 decisions to shift from bonds to stocks and back to bonds. The next section presents RAPM and investing in assets either weighted differently (e.g., 40-percent to a proxy liability, represented by a portfolio of assets that in 1999, before declining to 37.1 percent in 2011. Doeswijk et al. (2012) also examine the allocation to equities, government bonds, nongovernment bonds, and real estate during 1959–2011. They found that equities started at 51.2 percent in 1959, rose to 64.1 percent in 1968, and were at 63.2 percent in 1999, before declining to 37.1 percent in 2011.

In summary, the case for a relative asset paradigm focuses on a representative investor that seeks to invest assets relative to a proxy liability, represented by a portfolio of assets that mimic the projected required cash flow. The proxy liability provides a reference point, and it has a clearly measurable threshold return and valuation, with non-zero volatility; the zero-risk asset relative to this proxy is the proxy portfolio itself. Investing in assets either weighted differently (e.g., 40-percent stocks and 60-percent bonds versus a simple 60/40 benchmark) or with totally different assets (e.g., small-cap stocks) carries relative risk. Further, the PT hypothesized profile of risk aversion seems more closely linked to funded status than to plain equity valuations. The next section presents RAPM and attempts to show how it can reconcile core aspects of PT and APT within an MPT framework of relative asset pricing. This RAPM approach provides a different take on the treatment of stocks and bonds, given the role of bonds in global markets and portfolios. It also uses AMH concepts to possibly explain the shift from bonds to stocks and back to bonds.

The Relative Asset Pricing Model

In the interest of simplicity, we consider Step 1 decisions regarding principals and liabilities (figure 3) in describing RAPM. We also examine some phenomena that potentially are caused by Step 2 relative decisions. Muralidhar and Shin (2013) use an extremely simple model with liabilities to highlight how focusing on skill of agents helps establish a demand function for various assets that is different from CAPM.

We use (1) a simple, single-period, MPT-type model that replaces maximizing wealth (or assets) as the core objective with maximizing funded status (or assets/liabilities), allowing us to account for liabilities as a reference point. We also use (2) standard relative utility models in the vein of Abel (1990), Reisman and Lauterbach (2002), Cornell and Roll (2005), and Brennan and Li (2008), but we replace their reference points with our liability or liability proxy.

Define $M'$ as the market portfolio of all assets including risky and risk-free assets, $M$ as the market portfolio of all risky assets (as in MPT), $L$ as the liability portfolio, $F$ to be the traditional risk-free asset, and $w$ to be the market value weight of $M$ in $M'$, i.e., $r(M') = w \times r(M) + (1 - w) \times r(F)$. If the net aggregate supply of the risk-free asset is zero, then $w=1$, and $M=M'$. Recall that relative to $L$, $F$ is a risky asset even though it has no volatility or correlation to other assets/securities, so it is included in $M'$, but not $M$. The net result of using the MPT framework and relative utility models gives the asset pricing formulation under RAPM as in equations 6a, 6b, 6c, and 6d, each of which presents the same information.

We present the model in four different ways to make it easier for the reader as we cover different points of comparison to MPT, APT, and PT. For the details of how this model is developed, see Muralidhar, Ohashi, and Shin (2013, 2014a, 2014b).

$$E[r(i)] - r(F) = \frac{\text{cov}(r(M') - r(L), r(i))}{\text{var}(r(M') - r(L))} \times E[r(M') - r(L)]$$  \hspace{1cm} (6a)

Or expanding the premium term on the right hand side of equation (6a) gives

$$E[r(i)] - r(F) = \frac{\text{cov}(r(M') - r(L), r(i))}{\text{var}(r(M') - r(L))} \times E[r(M')] - \frac{\text{cov}(r(M') - r(L), r(i))}{\text{var}(r(M') - r(L))} \times E[r(L)]$$  \hspace{1cm} (6b)

Or rewriting the numerator of the “relative beta”/coefficient term in equation (6a) gives

$$E[r(i)] - r(F) = \frac{\text{cov}(r(M'), r(i)) - \text{cov}(r(L), r(i))}{\text{var}(r(M') - r(L))} \times E[r(M') - r(L)]$$  \hspace{1cm} (6c)

Or expanding for $M$ in equation (6a) gives

$$E[r(i)] - r(F) = \frac{\text{cov}(w \times r(M') - r(L), r(i))}{\text{var}(w \times r(M') - r(L))} \times E[w \times r(M') + (1-w) \times r(F) - r(L)]$$  \hspace{1cm} (6d)

Some quick observations highlight the appeal of RAPM in unifying some of the other major theories. First, this is effectively an MPT model, except it is in relative space. In relative space, investors prefer assets with high positive correlation to the liability because they better hedge the liability, and this is reflected in the (modified) relative beta coefficient in equation 6c. Also, as shown in equation 6d, in RAPM the risky assets’ market risk premium is expressed as the expected excess return over liability return. Unlike traditional CAPM, the market value weight of risky assets vis-à-vis risk-free assets matters in relative space, as shown in equation 6d. This is because in relative space the investor’s portfolio will be composed of allocations to the liability portfolio, the risky-asset market portfolio, and the risk-free asset. Therefore, the relative weights of the risky-asset market portfolio matters. Traditional CAPM is easily retrieved by assuming the liability, $L$, is deterministic with $r(L) = r(F)$ satisfying the no-arbitrage condition. This also results in a relatively simple formula for expected returns, much like MPT, though the devil is in the details about the actual proxying of the market portfolio and the liability portfolio. Following Black (1972), in RAPM, the liability/benchmark could serve as the zero-beta asset.

Second, RAPM has similarities to two-fund separation in that, in an equilibrium model, investors should engage in three-fund separation among cash, a liability hedge, and a return-seeking portfolio—which is exactly what investors are doing today, so theory now tracks practice—as shown in equation 6d. Third, it uses a reference point—liabilities—as
desired by PT. Fourth, equation 6b suggests that, even in a simple single-period model, the scope for additional factors depends on how the global liability portfolio is proxied. Unlike the market portfolio, the liability is more complex and can allow for many APT factors, discussed below. Fifth, changing regulations and evolving investor sophistication and reaction to market crashes have affected the liability proxy over the past five decades and levels of permitted/desired relative risk. Adapting RAPM to an intertemporal setting, future research might easily capture these phenomena. For example, after the crash of 2000–2002, many if not most pension funds went from being overfunded to underfunded, and this changed their risk appetite and desire for an equity risk premium, as suggested in figure 4. Also, as the regulators tightened the permissible relative risk, the SAA was modified (e.g., extending duration and increasing allocation to bonds), but relative risk budgets also tightened significantly.

At a high level, we have demonstrated how a simple model, represented in four different ways as in equations 6a, 6b, 6c, and 6d, could help unite many of the core concepts of various asset pricing theories. Next, we dig in to validate specific claims for MPT, PT, APT, and AMH.

Modern Portfolio Theory
RAPM uses the standard mean-variance model, where utility is based on relative assets (or the ratio of assets/liabilities or funded status) and where the liabilities are proxied with financial assets.16 Traditional MPT assumes that the risk-free asset is the reference portfolio and this is true for asset-only or absolute-return focused investors. As a result, one can see CAPM as a special case of the more general RAPM. RAPM preserves the key attractive feature of CAPM: It provides clear prescriptions for potential returns and reasonable asset allocation recommendations. In the “Implications” section, we discuss how an appropriate risk-adjusted performance measure can be developed from a specific application of RAPM.

Prospect Theory
PT has detected many behavioral biases that would be hard to reconcile with a simple RAPM. But it’s difficult to ignore that PT and RAPM share a tangible reference point that is connected to investor behavior: liabilities. This provides a base to test PT empirically as shown in figure 4.17 Because the later PT framework of mental accounts allows each mental account to have a unique liability proxy, it is easy to specify and formalize RAPM into the mental accounts approach.

Arbitrage Pricing Theory
Equation 6b makes a case for at least one additional factor, liabilities, other than the traditional market portfolio, and the “betas” are likely to be different from the CAPM betas.18 As discussed above, liability proxies (or SAAs) are typically linear combinations of a host of indexes/asset classes rather than being one simple instrument, portfolio, or asset class. Hence equation 6b might expand to equation 5 if each of the indexes in the global liability portfolio is uniquely expressed. To explain Fama-French factors, one can revert to the evolution of investment approaches to note that through the late 1990s, most funds proxied their liabilities via their SAA with portfolios composed of equities (typically large-cap and growth) and short-duration bonds. So it is possible the value and size effect reflect the fact that the liability in our equation gave a premium to value and small-cap stocks (because they are excluded assets) and large and growth stocks would require a lower return because of their liability hedging property.

One can make a similar case with the results of Chen et al. (1986) and Duarte (2013). Any difference terms (e.g., BAA minus government yields) can be split up as suggested earlier for Fama-French factors, with the first term associated with the risky portfolio, and the term with the negative coefficient associated with the liability proxy. Moreover, most liabilities, especially in pension funds, have a component tied to economic growth/industrial production because pensions depend on wages (likewise, most endowments have a spending policy that assumes 5-percent growth). Inflation is typically a component of pensions, too. The fact that fixed-income terms are additional factors syncs back to the fact that most pension fund liabilities can be replicated by a long-duration bond. These concepts will be pursued in future empirical research.

Momentum is a factor that requires explanation outside RAPM. Our explanation derives from investor behavior, because most funds had rebalancing strategies around the liability proxy/SAA, to account for the relative risk caused by drift away from the SAA. In effect, rebalancing strategies are mean-reverting and counter-trend, in that you sell the winning asset and buy the losing asset in order to rebalance. The momentum effect probably captured this tendency of investors to rebalance and this is validated in Sharpe (2010), as he demonstrates how using a momentum strategy when everyone else is counter-trending can be profitable. Barrett et al. (2011) argue for more intelligent rebalancing to account for the lost opportunity in traditional rebalancing policies, and momentum is one strategy in the recommended approaches.

Adaptive Markets Hypothesis
The more difficult reconciliation is to AMH, which is a more qualitative approach so we have no formulas or value functions to reconcile to. The core argument of AMH is that one needs to capture how investors adapt to changing markets, regulations, and opportunities. One classic behavior we note is that investors tend to change their SAAs over time to adapt to changing market conditions and opportunities. So the decline in small-cap effect and value effect is probably caused, in large part, by investors adapting and including alpha-generating assets in their liability proxies. As these investments became more liquid and replicable with passive indexes, they were incorporated into SAAs of funds. Also, as noted above, regulations changed post 2000, especially with the passing of the Pension Protection Act of 2006 in the United States.
and stricter regulation in Denmark, the Netherlands, and the United Kingdom. The secular decline in long-term interest rates also could explain this adaptation and one can expect changes in the demand for bonds, as regulators are changing requirements (e.g., MAP 21 in the United States and other smoothing approaches elsewhere).

Finally, we shed light on a strategy that is gaining traction: the low-volatility strategy. As Baker et al. (2011) note:

[C]ontrary to basic finance principles, high-beta and high-volatility stocks have long underperformed low-beta and low-volatility stocks. This anomaly may be partly explained by the fact that the typical institutional investor’s mandate to beat a fixed benchmark discourages arbitrage activity in both high-alpha, low-beta stocks and low-alpha, high-beta stocks. … Indeed, as mentioned earlier, this anomaly gained force over a period when institutional management in the United States went from 30 percent to 60 percent.

What Baker et al. (2011) propose is that, because of limits to leverage in delegated assignments, agents have an incentive to bid up the price of high-beta stocks and ignore low-beta/low-volatility stocks, thereby driving these phenomena in the past. The focus on agents is essentially Step 2 of our RAPM process, as shown in figure 3.

As the industry adapts to this opportunity, the low-volatility anomaly will disappear. We suggest this phenomenon is caused by the use of the Sharpe ratio or information ratio as a measure of risk-adjusted performance. As Modigliani and Modigliani (1997) note, if investors do not normalize returns for volatility, managers have an incentive to increase volatility in outperforming a benchmark. They demonstrate cases where a good manager may underperform the benchmark with low risk, and the investor can capture this manager’s skill by borrowing money and leveraging the allocation to have the same volatility as that of the benchmark. By contrast, a manager who beats a benchmark (i.e., positive information ratio) with naïve leverage should not be hired. The next section addresses this issue of appropriate risk-adjusted performance measures and how to preclude opportunities that play on volatility scaling to generate alpha; this is something a smart investor should be able to implement without incurring any cost.

**Implications**

Having demonstrated how a RAPM framework, anchored in how investors make decisions relative to liabilities or benchmarks, might reconcile many of the seemingly contradictory evidence vis-à-vis MPT, we now discuss the implications for asset pricing, asset allocation, rebalancing, and risk-adjusted performance.

**Asset Pricing**

Equation 6a may appear only marginally different from equation 1, but equation 6a (or even equation 6b) is more useful here because it highlights two key differences from CAPM. The most interesting difference is that RAPM, all else being equal, has an additional term (liability), relative to CAPM, that has a negative coefficient. Assets that provide a liability hedge, everything else being equal with respect to the market portfolio, will earn a lower return than those that do not correlate as highly with liabilities. Duarte (2013) reports findings that possibly fit this paradigm in that “investors are willing to accept lower unconditional returns when holding securities that are good hedges against inflation.” Also, if we assume that bonds (with volatility) are the liability replicating asset, then a clear case exists for why, over the long term, one can use the traditional building-block approach to expected returns.

In this approach, most investors start with inflation and add a real cash rate for the nominal cash return forecast, a real bond premium for bonds, and then a real equity premium for equities. Muralidhar (2001) highlights how institutional investors develop these forecasts and CAPM/MPT do not allow for this degree of granularity.

The other major difference is that the single beta in equation 1 is now replaced with two betas, which makes RAPM a bit more complicated than CAPM, with the added twist that the two RAPM betas are different from the CAPM beta (equations 6b, 6c, and 6d). This may explain why earlier investigations found that the positive relation between beta and average return was too flat, or the rejection of the premium per unit of beta being the expected market return minus the risk-free interest rate. In a simple way, RAPM requires a three-dimensional analysis of returns, volatilities, and correlation, and previous tests were largely two-dimensional (Muralidhar et al. 2014a).

In RAPM, the market risk premium is expressed in terms of excess return over liability return, where the market portfolio is defined as a market value-weighted portfolio of all assets including the risk-free asset. Traditional CAPM is the special case of RAPM where the liability portfolio return is deterministic, yielding a risk-free rate. In this case, the market risk premium is expressed as the risky assets’ market portfolio return over the risk-free return (equation 6d).

**Asset Allocation and Rebalancing**

MPT’s two-fund separation was an appealing concept, but it was not seen in many portfolios. RAPM suggests three-fund separation among cash, a liability hedge portfolio, and a risky (ex-liability) portfolio, and this is found increasingly in pension fund portfolios (Amenc et al. 2010). This approach also suggests that many funds need to first find which portfolio closely mimics liabilities, as opposed to an SAA that may have been derived from Markowitz’s MVO. It may be the case that liabilities for an endowment are less bond-like than those for a regulated corporate pension fund, but liability replication with financial assets is a critical first step in any asset allocation process.

We ignore Sharpe (2010) and Barrett et al. (2011) and their critiques of naïve rebalancing to focus on rebalancing...
between the liability hedge and a risky portfolio. Muralidhar (2001) suggested a simple view-neutral rule to conduct such a rebalancing based on the investor’s funded status. The ABN AMRO pension fund effectively applied a variation of this rule in 2008, making it one of the few pension funds globally to protect its funded status above 100 percent during the market downturn; the ABN AMRO case study is presented in Muralidhar (2011). In effect, for overfunded pension systems, the idea was to increase the liability hedge and lower the allocation to risky assets as funding declined, thereby serving as an asset-liability put. The reverse would be true for underfunded pensions subject to the caveat that the sponsoring company had deep pockets to sustain additional contributions if risky assets underperformed. Clearly, more informed rebalancing between $M$, $F$, and $L$ can be devised from RAPM.

One alternative asset allocation framework, especially for an underfunded pension plan, is to explicitly incorporate the PT-type utility with a reference point set by regulators or by pension participants. According to the standard concave utility, deeply underfunded pension plans’ asset allocations would be invested mostly in the liability portfolio, which may not be consistent with what pension participants desire because the underfunded status can never be improved without additional contributions. This implies that pension participants may have a PT-type utility with a specific reference point, at which point they have to inject additional contributions. By explicitly incorporating the locally convex characteristics of pension participants’ utility, asset allocation decisions can be better framed. This would make for a rich research agenda.

**Risk-Adjusted Performance**

We adopt the Modigliani and Modigliani (1997) approach to risk-adjusted performance because it expresses the output in percentage returns (making it easy for practitioners, because ratios do not say anything about performance), adjusts for differences in volatility, and provides investors with advice on how to structure portfolios for any asset or investment opportunity to ensure the highest risk-adjusted performance. Muralidhar (2000) argues that although Modigliani and Modigliani (1997) and the Sharpe ratio apply to CAPM investors, additional normalizations are needed in a relative paradigm. Two investment opportunities can have the same target relative risk to a benchmark or liability but can differ in volatility and correlation to the benchmark and, as a result, skill of the manager (using Ambarish and Seigel 1996). The Modigliani and Modigliani (1997) normalization adjusts for differences in volatility and so Muralidhar (2000) demonstrates how rebalancing across cash, the risky opportunity, and the benchmark (e.g., liability) can normalize for both differences in volatility and correlation. This gives risk-adjusted performance measures that keep all the attractive aspects of Modigliani and Modigliani (1997) but also provides ranking of opportunities that are identical to rankings based on skill. Ranking opportunities identically to those based on skill is important because in a principal-agent arrangement, principals would be expected to want to hire agents who are skillful, not lucky.

**The Path Forward**

This paper attempts to provide a broader tent under which many aspects of asset pricing models can be accommodated by examining investor behavior using a relative/liability-centric perspective, because this is how investment decisions are made. At a relatively superficial level, we show how RAPM can capture aspects of MPT, PT, APT, and possibly even AMH. While this attempt may be criticized for being simplistic, it suggests new paths for research even if researchers seek to cling to individual silos. For example, RAPM may be expanded into a multi-period model to develop an intertemporal model (à la Merton 1973), or to introduce heterogeneous investors with different liabilities and risk aversion, and to even develop a two-stage model à la Van Binsbergen et al. (2008) to examine the impact of agency on asset prices in this framework.

Within PT, this approach begs the question as to whether the original Kahneman and Tversky (1979) research asked participants the appropriate questions. For example, investment behavior is more about achieving objectives (servicing liabilities) than about arbitrary lotteries, and one wonders how the responses of individuals without investment experience would differ when compared to those of institutional investors. Further, would responses differ if respondents were asked to anchor their decisions with a clearly articulated liability, as opposed to the traditional questions posed by Kahneman and Tversky (1979), which did not anchor the investment choice to any liability considerations? Our initial tests in this area, using the same approach as Kahneman and Tversky (1979), suggest that funded status matters for certain parts of the risk spectrum and the behavior of underfunded participants is different from what they discovered. The mental accounts research suggests that an individual could implicitly consider any one of many mental accounts in evaluating an investment opportunity; this is fertile ground for new PT research to examine if decisions change for explicit mental thresholds.

Even with APT, this approach provides a robust justification for factor choice, depending on the composition of the world liability portfolio. Therefore, rather than using existing findings and postulating on how they might fit RAPM (as in this paper, because of how liabilities were proxied), an interesting research avenue would be to collect this data historically to the extent possible (a nontrivial task) and then start to test for factors.

In summary, this paper provided a RAPM that, though not unique because other researchers have considered relative utility functions and models in the context of peers and agency issues relative to equity or consumption benchmarks, is different because it anchors the asset pricing/allocation model to a more general setting that all investors can relate to: liabilities. This simple approach provides new recommen-
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1dations for asset allocation, asset pricing, and risk-adjusted performance calculation. We demonstrate how CAPM is just a special case of this more general model, and how RAPM can explain various seemingly retrograde aspects of competing theories. In effect, this requires a repositioning of asset pricing theory from an equity (or market portfolio) centric view to a liability-centric view, where the liability can be a complex mix of assets and factors. To quote Copernicus (1543), “Since the sun remains stationary, whatever appears as a motion of the sun is really due rather to the motion of the earth.”

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Endnotes

1 As Fama and French (2004) note with respect to Merton (1973) and Ross (1976), “[A]dding diversified portfolios that capture covariation in returns and variation in average returns left unexplained by the market is in the spirit of both the [Merton’s intertemporal] ICAPM and [Ross’s] APT.”

2 An exception would be Elton et al. (1995), which focuses on using APT approaches to explain bond fund performance. Bonds pose a problem because they age and mature, so it is easier to test for appropriate factors on bond funds that have a somewhat stable duration or focus.

3 The market portfolio is a theoretical bundle of investments that includes every type of asset available in the world financial market, with each asset weighted in proportion to its total presence in the market. In most of the empirical research, this portfolio is proxied with a portfolio of risky stocks. Stambaugh (1982) highlights the difficulty in establishing this portfolio. In its purest form, the market portfolio should include financial assets, consumer durables, real estate, and human capital.

4 The Treynor ratio is similar to the Sharpe ratio except that the denominator is beta, not volatility.

5 The Modigliani and Modigliani (1997) variation of the Sharpe ratio suggests that the information ratio can be generated by strictly leveraging the benchmark, so it recommends calculating risk-adjusted performance after normalizing for differences in volatility. The Modigliani and Modigliani (1997) measure provides rankings that are identical to the Sharpe (but different from those on the information ratio). The key differences between the two approaches are (1) that the Sharpe is a ratio and the Modigliani and Modigliani (1997) approach reports the data in percentage return terms; and (2) the Modigliani and Modigliani (1997) measure provides useful information on how to create a risk-adjusted portfolio using leverage/deleveraging (in an argument reminiscent of the Modigliani-Miller theorem). Modigliani and Modigliani’s volatility-normalized rankings provide useful information about whether leverage or true asset selection contributes to performance and also how portfolios should be levered/delevered.

6 The result of these assumptions is that it introduces a “regret” property into the utility, which effectively increases the degree of “effective risk aversion” because losses dramatically impact utility. The primary goal of Barberis et al. (2001) is to “help explain the high mean, excess volatility, and predictability of stock returns, as well as their low correlation with consumption growth.”

7 If viewed from the mental accounts framework, every portfolio has its own stream of desired cash flows that needs to be serviced and that in turn is the liability for that mental account. One could similarly make a case for identifying a “liability” for endowment, insurance, or other institutional portfolios.

8 In reality, principal-agent relationships often have multiple and overlapping layers, but we ignore that for now. For example, a pension fund has a chief investment officer and an investment team that would be the agent in the strategic asset allocation-setting phase. The internal staff then hires external managers and the external managers are the agents in that relationship. So in a funny way, the internal staff is both agent and principal and this creates a number of issues that are beyond the scope of this paper.

9 In the early 1990s, one of the authors visited many funds in the United States that would perform a simple MVO and select the SAA with no connection to the projected liability except a target return, even though many researchers had argued for surplus optimization (see Sharpe and Tint 1990; Leibowitz 1986; Leibowitz and Henriksson 1988). Therefore, the optimal MVO portfolio became the unintentional liability proxy. Rauh and Novy-Marx (2011) have been criticizing U.S. public pension funds for using a high discount rate because of GASB rules for public funds. The implication for our approach is that public pension funds have effectively proxied liabilities with a mix of stocks and bonds, while corporate pension funds, because of regulations governing corporate pension funds, currently are proxying liabilities with long-duration bonds with a credit component.

10 Arun Muralidhar was responsible for the SAA of a large global pension fund from 1995–1999; liabilities were considered in the SAA process, but MPT approaches were used extensively, leading to similar issues.

11 Van Binsbergen et al. (2008) focus on this two-stage delegation. Their focus is a bit different from this paper’s focus, but they find that because of this two-stage delegation “the CIO’s uncertainty about the managers’ risk appetites increases both the costs of decentralized investment management and the value of an optimally designed benchmark.” Cornell (2009) discusses skill, but the authors of this paper find the best theoretical and robust measure of skill to be Ambarish and Seigel (1996).

12 See http://www.publicfundsurvey.org/publicfundsurvey/summaryoffindings.html. Figures A and B in this publication show how the funded status (A/L) has changed over the years. Figure O shows the
change in the asset allocation over the same window and one can see how bonds have declined and alternatives have increased (i.e., risk aversion has declined as funded status fell). See also endnote 17 on the responses of institutional investors to a risk attitude survey.

Many pension funds are reportedly underfunded because accounting treatments force such calculations, and many endowments have experienced equivalent problems and had to lay off faculty, cancel the construction of buildings, and borrow large sums of money to meet commitments. The liabilities of these endowment investors are less explicit and hence our use of pension funds as the representative investor, but again one can read the SAA as the proxy liability assuming that the SAA is properly set by the principal.

The appendix discusses the impact of using funded status (assets divided by liabilities) versus surplus (assets minus liabilities). Both approaches incorporate liabilities and provide a notion of “relative wealth.”

An investor’s portfolio will be composed of (1) a liability portfolio whose market value is equal to asset value (in case of funded ratio utility), or to liability value (in case of surplus utility), (2) a risky-asset market portfolio, and (3) risk-free assets. The RAPM in this paper is a “funded ratio utility” version of the model. The “surplus utility” version RAPM will have an additional parameter; namely the initial funded ratio.

Had we used maximizing surplus as opposed to maximizing funded status as the objective function, RAPM would be modified slightly to include the initial funded status, but would still incorporate liabilities in the asset pricing model. See appendix.

In preliminary and unpublished research conducted with students at George Washington University and employees at Cambridge Associates, Kaheman and Tversky (1979) questions were presented to participants to evaluate whether they were influenced by initial wealth or initial funded status in their risk-taking behavior. From a preliminary analysis of the data, it appears that when participants are underfunded, their behavior is different from how they behave if they were not given any information on liabilities (i.e., behaved based solely on their wealth). This will be explored in future research but suggests a role for liabilities as the reference point.

Similarly, in preliminary and unpublished research on the risk attitudes of chief investment officers (CIOs) of global institutional portfolios (with responses from Canadian, Dutch, Latin American, New Zealand, and U.S. investors), maximizing funded status seems to be the dominant objective. Twenty-nine CIOs responded to the survey (out of 100 polled) at the time of publishing this article (34 percent from public pensions, 31 percent from corporate pensions, 24 percent from endowments/foundations, and the balance from foreign-exchange reserves, Taft-Hartley plans, etc.). Sixty-two percent subject identified maximizing funded status as the primary goal; 14 percent identified maximizing absolute returns and 7 percent identified maximizing assets. There was more dispersion in their primary risk measure (14.29 percent picked volatility of contributions, 35.7 percent picked portfolio volatility or Value-At-Risk, 25 percent picked tracking error relative to a liability or SAA, and 25 percent picked other criteria, including drawdowns/permanent loss of capital, surplus risk, and no single measure).

Fama and French (2004) point to Merton’s (1973) intertemporal CAPM where a case can be made for an additional factor to hedge the intertemporal changes in the opportunity set. RAPM gives an additional factor in a single-period model.

Notice in equation 6d, the coefficient for \( r(M) \) is conditioned by \( w \), which under reasonable circumstances would lie between zero and unity. Similarly, an additional term in \( r(F) \) is conditioned by \( 1 - w \). Even ignoring the impact of \( r(L) \), this facet of the RAPM equation could provide some value in explaining the higher intercept and flatter slope in CAPM tests.


References


**Appendix: Funded Ratio vs. Surplus**

Two competing measures define relative space: surplus (i.e., excess return) and funded ratio (i.e., relative return index). Sharpe and Tint (1990), Leibowitz and Henriksson (1988), Rudolf and Ziemba (2004), and Cornell and Roll (2005) consider surplus as a relative measure, whereas Reisman and Lauterbach (2002) and many pension regulators consider the...
ratio as the relative measure. RAPM assumes the funded ratio is a relative measure and here we show how the two different measures impact investors’ asset allocation decisions. Muralidhar et al. (2014b) develops RAPM based on both surplus and funded status.

Denote \(A_0\) and \(L_0\) as the asset and liability values at time 0, respectively, and \(r(A)\) and \(r(L)\) as the respective asset and liability returns. In the case of funded ratio optimization, the future funded ratio can be expressed as \([A_0 \times (1 + r(A))] / [L_0 \times (1 + r(L))] = (A_0 / L_0) \times [(1 + r(A)) / (1 + r(L))].\) Therefore, the time 0 funded ratio plays the same role of initial wealth in the asset-only optimization case. For log utility, for example, initial wealth does not matter in asset decisions, so in RAPM, using log utility, initial funded status will not matter in portfolio decisions. In the case of surplus optimization, future surplus can be expressed as \(A_0 \times (1 + r(A)) - L_0 \times (1 + r(L)) = A_0 \times [(1 + r(A)) - (L_0 / A_0) \times (1 + r(L))].\) Therefore, the inverse of the initial funded ratio, \(L_0 / A_0\) acts as a scaling factor on liability portfolio return. For log utility in RAPM using surplus, for example, the initial funded ratio will matter in asset decisions.

The two competing measures will result in different holdings of the liability portfolio. If, for example, \(A_0\) and \(L_0\) are $0.5 billion and $1.0 billion, respectively, the liability portfolio holdings will be $0.5 billion and $1.0 billion for the funded status and surplus approach, respectively. In other words, the liability holding amount for the funded ratio case will be the same as the asset value, whereas it will be the liability value in the surplus case. The RAPM in this paper does not have any funded ratio component because the relative measure is defined as the funded ratio. If, instead, it is defined as surplus, then the initial funded ratio will appear in the RAPM equation. This will be explored in future research.