Portfolio Construction and Rebalancing for Individuals

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There is a long-held view that portfolio decisions can be made without regard for the workings of the asset owner, be that a firm or an individual. Embodied in Irving Fisher’s separation theorem,1 this view makes the portfolio construction process clean and generic. Combined with the assumption of a normal return distribution, it leads to the mean–variance approach that has dominated finance.

The mean–variance approach to portfolio construction is free of context. Mean–variance doesn’t account for whether the asset owner is someone starting out in a career or a pensioner, a college endowment or a nonprofit, wealthy or not, with a long-term horizon or not. The portfolio manager is given the level of risk and the benchmark portfolio and then is off to the races. Add the capital asset pricing model and even the benchmark is all but determined.

For an individual, this doesn’t work. An individual’s portfolio and benchmark cannot be separated from personal objectives. Risk preference cannot be pinned down to one number. Mean–variance specifies a linear trade-off between return and risk, and risk, in a normally distributed world, is sufficiently described by the variance of returns. But for an individual, this linear relationship is displaced by a multidimensional set of objectives, each with its own market factor weights and distributional characteristics. These vary from one individual to another, and they vary for any one individual during a lifetime.

That is, when we move portfolio construction into the realm of the individual, things get personal and become dynamic.

The starting point of the path that takes portfolio construction into the human realm is mean–variance optimization (MVO), which, with various flavors, harkens back to Nobel laureate Harry Markowitz. The next step is to reframe the optimization problem in a way that can speak to an individual’s objectives. This means moving the portfolio design from asset space to factor space. The objectives can then be categorized into buckets such as security, lifestyle, and aspirational goals. This is more than taxonomy. The objectives are distinct, even going so far as being sourced from different parts of the brain.

This approach reframes portfolio construction as multidimensional and dynamic, and also as risk aware. The objectives are couched in terms of the sources of risk, the acceptable levels of these risks, and more than that, their distributional characteristics.

THE STARTING POINT: MEAN-VARIANCE OPTIMIZATION

MVO is the standard tool for constructing and managing portfolios. It formalizes portfolio construction’s problem as the search for the blend of assets that achieves the maximum possible return subject to a constraint on the variance of returns.

The MVO problem is an optimization problem. The objective is to maximize returns from the asset allocations in the portfolio. The mathematics of this optimization is described in detail in appendix 1.

MVO captures the basic characteristics of the portfolio construction process, but it faces some well-known limitations or instabilities when put to task on real-world investment decisions.

One of these limitations comes from the sensitivity of the MVO framework to expected returns. Another comes from uncertainty inherent in the risk-return paradigm of MVO. These limitations also are described in detail in appendix 1.

At one time or another, we all have seen the practical implications of these limitations. We were at a hedge fund some years back when we had our first run-in with these instabilities. We loaded a portfolio’s positions into an optimizer, pressed the button, and found out that 25 percent of the portfolio should be in General Mills. The reason, as we discovered, was that during two short periods and for idiosyncratic reasons, General Mills moved in the opposite direction of the market. The odds of that happening again are low, but as far as the computer doing the optimization is concerned, that is the way the world unfailingly works.

For larger portfolios with hundreds of assets, typical for separately managed accounts, MVO needs to be used with even more caution. It has been shown that for typical financial portfolios of
500 assets, e.g., the underlying stocks of the S&P 500, insufficient caution can lead to a severe underestimation of risk (Laloux et al. 2000).²

**PORTFOLIO CONSTRUCTION EXTENDED TO MULTI-DIMENSIONAL OBJECTIVES**

Even if these issues with stability were moved out of the way, the notion of risk within optimization deserves to be further scrutinized. Investors are treated as if they boil risk down to one number, usually the volatility of returns, and that they care about what is held in the portfolio only to the extent that the total level of risk meets an objective. But for individuals, there is more to risk than the volatility of returns. Risk is multi-dimensional and ultimately qualitative. It is not a number, a single constraint in an optimization.

This means that the details of portfolio construction matter. Optimization must dig deeper, take more information into account than the expected returns pulled from capital market assumptions and the standard deviation coming out of a variance-covariance matrix. Some portfolios will have more sensitivity to a particular industry or geographic region. They also might have biases toward portfolio construction in terms of dividends or a company’s size.

These differences can matter to the individual. Treating risk as a single statistic, the standard deviation of return—or any other number, for that matter—removes the personalization that an individual requires. An advisor might set up the portfolio for a particular goal, and different parts or subsets of the portfolio might have different mandates. Focusing on the overall mark-to-market volatility of the portfolio distracts from a rich understanding of the drivers of the risk.

The first steps in moving beyond the standard approach to portfolio construction are understanding the drivers of risk and setting up a framework that incorporates them into the optimization and portfolio construction process, not only allocating across asset classes but also by making investment decisions by sectors, sub-sectors, styles, or the duration of fixed income exposures.

In a similar vein, just as we allocate to assets or asset classes, we can choose to allocate a total amount of risk across asset classes. The risk budget can be something as basic as volatility or something more involved such as expected shortfall. The core idea then is to allocate a fixed amount of risk among the various assets within a portfolio just as we would have allocated a fixed amount of capital to those assets. Budgeting the risk in such a way provides the first step toward a risk-first approach to asset allocation.

A major advantage of this approach is that the mathematical formulation removes the hurdles of the uncertainty in risk-return as well as the sensitivity to the return assumptions. We are then no longer in the realm of MVO but within the realm of risk allocation. The problem can nonetheless be formulated in terms of an optimization program that, given the risk allocations to the various assets and asset classes, will produce an asset allocation that best matches the risk allocations to risk factors (Bruder and Roncalli 2012). Mathematical details of this formulation are shown in appendix 2.

The problem of risk allocation is independent of whether we are allocating to factors or to assets. But allocating risk to factors is nonetheless advantageous because of the following:

- Risk factors are more stable than assets, so correlation issues—although still there—are lessened.
- The factor influences thread through the assets, so the relationships among them are tethered to something more than their historical behavior.
- Most of the risk usually can be found in a handful of intuitive factors rather than grappling with perhaps hundreds of assets.

- Factors abstract away from the idiosyncrasies of the assets, allowing us to focus on what are the true drivers of the performance of assets. Therefore, portfolios that might look very different from the asset allocation perspective can nonetheless be compared at the factor level.

This optimization problem is now flexible enough to consider client preferences. These preferences can take many forms: as minimum/maximum bounds for specific asset classes, or restrictions on buying and selling of certain securities or asset classes. Clients also might have preferences on trading within specific accounts. Integrating such preferences within a factor risk allocation framework provides advisors the tools to customize the portfolio construction process to address each client’s individual needs.

**PORTFOLIO CONSTRUCTION HUMANIZED FOR A HIERARCHY OF NEEDS**

Brain scans show that the prospect of gains and the worries about losses—losses that really matter, so take lottery tickets off the table—light up different parts of our brains.³ Why? We can go back to the genes we have from our (successful) hunter-gatherer ancestors:

- A good, positive return outcome: Finding a patch of berries
- A bad, left-tail outcome: Getting killed by a lion

There is nothing that is as positive as getting killed is negative. The human brain’s reptilian complex is engaged with the risk of getting eaten. Berries don’t go there.

If a human brain puts various types of risk—taking decisions into different buckets, the approach to portfolio construction should do so as well.

One framework in this vein for formulating a client’s long-term investment needs is provided in Chhabra (2005).
In this framework, client objectives can be divided among three buckets: security, lifestyle, and aspirational. As Chhabra points out, the foundation for investing among these buckets is not asset allocation but risk allocation.

For the security bucket, the client requires a minimum level of wealth that provides a baseline level of financial security. The lifestyle bucket’s objective is to earn market returns and maintain a steady cash flow that preserves a target standard of living. Finally, the aspira-
tional bucket’s mandate is to allow the client to take substantial downside risk for an outsized payoff to meet goals such as buying a second home, contributing to charities, or bequeathing to children. These buckets have the ring of Maslow’s hierarchy of needs: protect, live, dream.

This approach recognizes that portfolio design faces risk objectives that are far beyond the scope of MVO. For the security bucket, it is minimizing downside risk; for the lifestyle bucket, it is having positions that do at least as well as the market; and for the aspirational bucket it is taking on riskier or more concentrated investments.

For the purposes of optimization, these three buckets have three different objec-
tive functions. The security bucket’s objective function is minimizing the maximum drawdown of the portfolio within a given period of time. At the other extreme, the aspirational bucket’s objective function is focused on generating high returns. The lifestyle bucket lies between the two and is the most amen-
able to a Markowitz–like framework.

An alternative way to formulate these buckets is to account for the asymmetry of returns. Whereas the MVO framework focuses on the variance of returns—and therefore treats positive returns the same as negative returns—this multidimen-
sional view leads us to confront the asymmetric nature of market returns: security focused on limiting the impact of negative returns, aspirational focused on maximizing positive returns, and lifestyle focused on tracking the market.

Although the security/lifestyle/ aspirational framework is a powerful one, how one should go about choosing assets that will address the specific requirements of each bucket is a crucial question. Investors have differing views about the level of downside risk they are willing to stomach and what they consider aspirational investments.

Also, within the asset space, the boundaries among the three buckets are rather fuzzy.

The key to the implementation of a security/lifestyle/aspirational framework is shifting perspective from asset allocation toward risk allocation. With this comes the next question: Should advisors allocate the risk in asset space or in factor space?

We propose an extension of the security/lifestyle/aspirational framework by using risk factors to delineate the tenu-
ous borders among investment buckets. The choice of risk factors to drive alloca-
tion among the various buckets is an intuitive one. Rather than focusing on the individual characteristics of assets and then allocating an asset to one bucket or the other, risk factors allow us to distill the essence of what drives the risk of each bucket.

For instance, the security bucket should have a blend of assets that have expo-
sures to value, quality, and liquidity factors. There will be a substantial allo-
cation to short-term Treasuries or other cash equivalents. The focus is more on low–risk assets rather than outright diversification. Similarly, the aspira-
tional bucket is focused more on the growth and beta factors. Alternative investments such as private equity and hedge funds lie within this bucket.

Finally, the maintenance of lifestyle can be achieved by having risk allocated across a wide variety of equity and fixed income factors. Within this bucket, the emphasis is on managing market risk and therefore the advisor can make tactical trades to and from a benchmark factor risk allocation.

PERSONALIZING PORTFOLIO DESIGN
When choosing from a selection of model homes, a salesman’s question will be: “Which of these do you like best?” But when an architect is hired to design a house from the ground up, the ques-
tion is: “How do you live?”

This is the difference between picking from prefab model portfolios and context–driven portfolio design. Turnkey asset management programs, for exam-
ple, construct model portfolios to choose from. Portfolio construction done with context allows an advisor to move from prefab to customized. Customization considers the values, life choices, and preferences of the client by building the portfolio from the ground up with the right weighting of factors and with the right set of assets.

The steps presented in this article sketch the process for doing this—moving from the standard MVO to restating the opti-
mization in factor space. These steps use that factor approach to address the two sides of the risk equation that matter to individuals: the portfolio on one hand and the personal risks on the other.

Contrary to the long–held approach of separating portfolio decisions from the activities of the individual (or firm), these are interactive and dynamic. If the portfolio underperforms, the client will weigh security more highly. If the client has an unexpected increase in income, the objectives will tilt more toward aspi-
ration and away from security or even lifestyle. This is more than a change in risk aversion. It is a change in the distri-
bution and the time frame of returns.

The factor risk allocation framework is flexible enough to accommodate the heterogeneity of client portfolios. Even when the advisor hasn’t set up the client’s portfolio within the factor–risk
allocation framework, the advisor can nonetheless set up model portfolios that take the client’s various financial objectives into account. These financial objectives can be classified into different buckets, with each bucket requiring a different blend of factor–risk allocations. Existing models can then be managed within the factor–risk allocation framework. As market conditions change, we can use the framework to rebalance the portfolio. Exposures of assets to individual risk factors are stable when markets are stable but adapt quickly when large changes occur. Rather than focusing on a fixed allocation target, advisors can track the factor–risk contributions and then rebalance client portfolios accordingly.

An investment portfolio is effectively a set of sub-portfolios, each with a particular mandate, possibly with underlying accounts that have different objectives. Clients might hold legacy positions or place constraints on the buying or selling of certain assets. Any portfolio construction or rebalancing exercise should at least be aware of these conditions. The investment management world needs tools that integrate the essentials of the client’s objectives and that allow for customization to a client’s specific investment needs. Today’s computing power, combined with advances in mathematical techniques, can help advisors evolve their toolkits to design portfolios that address the multidimensional needs of a heterogeneous set of clients.

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ENDNOTES

1. Investopedia defines Fisher’s separation theorem as an economic theory that postulates that, given efficient capital markets, a firm’s choice of investment is separate from its owners’ investment preferences and therefore the firm should only be motivated to maximize profits.

2. Results from random matrix theory reveal that the bulk of the information within a covariance matrix of such a portfolio is contained in only a handful of large eigenvalues, with much of the smaller eigenvalues being purely noise. Thus, the result of an MVO will be a portfolio where the highest weights are proportional to the least informative directions.

3. There is a wide set of research on brain function for decisions under risk; for example, see Ernst et al. (2002).

REFERENCES


APPENDIX 1: MVO

The MVO problem, briefly, is an optimization problem where the objective is to maximize a quadratic function of the asset allocations of the portfolio. With these sets of allocation, denoted by $x$ in what follows, we have the formulation:

$$\max_x \mu(x) - \frac{1}{2} \lambda x^T C x$$

(1)

Here $\mu(x)$ is the return of the portfolio, $C$ is the variance–covariance matrix of the $N$ assets, and $\lambda$ is the risk–aversion parameter. When $\lambda = 0$, then we seek the set of asset allocations that maximizes the return, whereas at the other extreme of very large values of $\lambda$, we seek an allocation that minimizes the total variance of the portfolio. It captures the basic characteristics of the portfolio construction process but faces well-known limitations when put to task on real-world investment decisions.

One of these limitations comes from the sensitivity of the MVO framework to expected returns. Errors in the returns are amplified through the optimization. For instance, capital market assumptions for returns, whether sourced through a third party or computed internally, come with sizable error bounds, and as one of the first studies on the robustness of MVO suggests, “Mean–variance optimizers are estimation–error maximizers” (Michaud 1989).

Another is the risk–return paradigm of MVO where the assumptions about the risk of assets are within the covariance matrix of asset returns. The optimal portfolio computed through MVO results in asset weights that are proportional to the inverse of the covariance matrix, which leads to small errors in the estimation of the covariance matrix to be magnified when computing optimal portfolio weights (Potters and Bouchaud 2020). Intuitively, we can see that an error of the order $\mathcal{E}$ in the estimation of the covariance matrix will result in optimal asset allocations that will differ by an order $\frac{1}{\mathcal{E}}$.

Conventional methods treat the elements of the variance–covariance matrix as a given, not as point estimates with a cloud of uncertainty around them. This is the source of much of the limitation because the computer takes the values as literally the nature of the
system and optimizes accordingly. If the correlations going forward are different—which they will be—the certainty with which the optimization applied the variance–covariance matrix will be wrong.

**APPENDIX 2: RISK ALLOCATION**

If our risk budget is determined in terms of the volatility of the portfolio, then the mathematical problem of risk allocation takes the form below.

$$\min_x \sum_{i=1}^{i=N} \left( \frac{x_i (C x)_i}{x^T C x} - b_i \right)^2$$

Here $b_i$ are the risk budgets for the chosen asset $i$ and $C$ is the variance–covariance matrix of the $N$ assets. Noting that the optimization problem strictly is speaking within the realm of non-linear programming—sequential least squares quadratic programming (SLSQP) (Boyd and Vandenberghe 2009). Performant algorithms exist for this class of problems and results can be computed quickly using modern computer hardware (Kraft 1988). It turns out that we get a second benefit from this: a move away from the dependence of the optimization on the inverse of the covariance matrix that leads to spurious and occasionally even absurd results.

From a strictly mathematical point of view, the problem of risk allocation as described in equation 2 is independent of whether we are allocating to factors or to assets. Allocating risk to factors is advantageous for the reasons listed in the text of this article.

$$f_k = \frac{\beta_k (C_F \beta)_k}{\beta^T C_F \beta}$$

Here $\beta_k$ is the exposure of the portfolio to the $k$th factor, and $C_F$ is the factor covariance matrix. Note that the exposures are computed for the whole portfolio, and therefore the $\beta_k$ above depend on the asset allocations. We can then write the factor risk allocation problem as:

$$\min_x \sum_{k=1}^{K} \left( \frac{\beta(x)_k (C_F \beta(x))_k}{\beta(x)^T C_F \beta(x)} - b_k \right)^2$$

In this formulation, we have made explicit the dependence of the factor exposures on the asset allocations $x$. The result of the optimization still is an asset allocation, but the risk contributions are defined through the factor exposures and the factor covariance matrix. The risk budgets now have the subscript $k$, highlighting that we are allocating the risk to individual factors. This reformulation from asset space to factor space doesn’t change the nature of the problem.

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