RETIREMENT MANAGEMENT JOURNAL

A reprinted article from Volume 10, Number 1, 2021

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CONSEQUENCES FOR DEFINED CONTRIBUTION AND DEFINED BENEFIT RETIREMENT PLANS

By Zvi Bodie, PhD

INVESTMENTS & WEALTH INSTITUTE®

VOLUME 10
NUMBER 1
2021

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ABSTRACT

It is widely believed that although stocks are very risky in the short run, in the long run they are far less risky and are sure to outperform risk-free investments such as government bonds. This belief is a dangerous fallacy. It leads to the illusion that one can earn an equity risk premium without bearing risk. In this paper, I first explain the flawed reasoning behind the fallacy: the appeal of the misleading concept of time diversification. Then I discuss the probability that an equity shortfall is indeed a decreasing function of the length of the investor's holding period *T*. But the severity of a possible shortfall, and therefore the cost of insuring against it, is an increasing function of *T*.

Finally, I explore some of the fallacy's harmful consequences with respect to (1) the regulations governing employer-sponsored retirement plans, such as 401(k) plans, that automatically enroll participants into qualified default investment alternatives (QDIAs) such as a life-cycle or targeted-retirement-date fund, a balanced fund, or a professionally managed account, and (2) the measurement and funding of state and local government pension promises, and I recommend measures to counteract them. These include (1) adding a new QDIA option in employer-sponsored retirement plans that automatically enroll plan participants in QDIAs and (2) the Government Accounting Standards Board (GASB) adopting new rules for measuring and reporting the liabilities of state and local government pension plans.

INTRODUCTION

It is widely believed that although stocks are risky in the short run, in the long run they are far less risky and are sure to outperform risk-free investments such as government bonds. This is a dangerous fallacy. It leads to the illusion that one can earn an equity risk premium without bearing risk. It violates the core principle of economics: TANSTAAFL—There Ain't No Such Thing as a Free Lunch. It implies that the stock and bond markets provide unlimited arbitrage opportunities. In this paper, I explore some of the harmful consequences of the fallacy with respect to (1) the regulations governing employer-sponsored retirement plans, such as 401(k) plans, that automatically enroll participants into QDIAs such as a life-cycle or targeted-retirement-date fund, a balanced fund, or a professionally managed account, and (2) the measurement and funding of state and local government pension promises.

I measure the risk of stocks as the market price of insurance against earning less than the risk-free interest rate. Such an insurance policy is equivalent to a European put option with strike price equal to the forward price of the underlying stock index. Both in theory and in practice, the price of such a "shortfall put" increases—rather than decreases—with the time to expiration, *T*.

Next, I explore two of the harmful effects that the fallacy has had in the past and continues to have in the present:

- Participants automatically enrolled into QDIAs are encouraged to take more market risk than might be appropriate.
- The funded status of state and local government defined benefit pension plans is, under current rules established by the Governmental Accounting Standards Board (GASB),[1] misreported. Actuaries, per GASB 67,[2] base, in part, the discount rate for measuring liabilities on the long-term expected rate of return on plan investments, justifying it on the grounds of a long time horizon. This "accounting arbitrage" results in reported liabilities that are too low. The result is underfunding of the true liabilities. Underfunding and the mismatch between the risk of plan assets and liabilities can lead to bankruptcy of the political entities sponsoring them.[3]

The final part of the paper considers policy measures to counteract the harmful effects of the fallacy. To deal with the first effect I suggest that employer-sponsored plans that automatically enroll plan participants be required to offer, in addition to the current choices, a guarantee of promised results, a guaranteed minimum benefit, as a QDIA option. The terms of the guarantee should be standardized so that participants can understand it. To deal with the second effect, I suggest that the rules as established by the GASB for computing the pension liabilities of state and local government defined benefit plans be changed and made consistent with the principles of modern finance.

THE TIME DIVERSIFICATION FALLACY

We begin by examining how one major commercial provider of investment advice explains to potential clients the reasoning behind the conventional view that stocks become less risky the longer the holding period. Below I present some excerpts from the Morningstar Investing Classroom website.[4]

> *Data on risk and volatility shows that the range of returns appears less volatile with longer holding periods.*
>
> *Over the long term, periods of high returns tend to offset periods of low returns. With the passage of time, these offsetting periods result in the dispersion of returns gravitating or converging toward the average. In other words, while returns may fluctuate widely from year to year, holding the asset for longer periods of time results in apparent decreased volatility. While the stock and bond markets can be risky in the short run, time has a moderating effect on market risk. The longer you hold a stock or bond investment, the lower your chances of losing money, and the greater the odds of earning a return close to the long-term average. For example, a one-year investment in stocks has historically produced returns ranging from +53.9% to −43.3%. Over ten-year periods, however, returns have varied from −0.9% per year for the worst ten years to +20.1% per year for the best ten years. As you can see, risk can be substantial over short periods. But over longer horizons, the chance of losing money is substantially reduced.*

The Morningstar reasoning is invalid. Samuelson (1997) proposed a simple and convincing way of demonstrating why.[5]

> *Here is how to test the theory. Write down all those 1,800 percentage changes in monthly stock prices on as many slips of paper. Put them in a big hat. Shake vigorously. Then draw at random a new couple of thousand tickets, each time replacing the last draw and shaking vigorously. That way we can generate new realistically representative possible histories of future equity markets.*
>
> *Is it true that in all these histories you always come out ahead in stocks rather than safe-but-less-volatile securities? Definitely not. Most of the time the buy-and-hold common stock investors do beat their more cautious neighbors; and, as the time horizon N becomes larger, the odds do grow that the bold holders of stock will win the duel. But it is also true that a longer time horizon brings bigger losses when an inevitable loss does occur.*
>
> *Canny risk averters should always keep in mind, in a rational, nonparanoid way, the pains they will feel in those probability-calculated bad-outcome scenarios. (Ask yourself: Will stepping down toward a poverty level, when that rarely but inevitably does happen, outweigh for me the pleasures that occur in those likely outcomes when my equity nest egg does increase?) When we each do that, those of us who truly are more risk averse will rationally hedge our bets by limiting our exposure to volatile equities.*

Although it is true that the average compound rate of return has less dispersion the longer the time horizon, it is not a sign of decreased risk because the average rate of return is not the variable of interest. The standard deviation of the average rate of return declines with the length of the time horizon because it is an average.

If σ is the standard deviation of the annual rate of return for one year, and T is the number of years to the time horizon, then the standard deviation of the average annual rate of return for T years will be $\frac{\sigma}{\sqrt{T}}$, assuming that returns have no serial correlation. Thus, the measured volatility declines steadily as a function of T.

But what investors care about is the dispersion of the wealth or lifetime income they will have at the end of T years, and not the average rate of return. Instead of dividing the annual standard deviation σ by the square root of T, we have to multiply by it. The standard deviation of final wealth equals the initial wealth times $\sigma\sqrt{T}$. As T increases, the probability distribution of terminal wealth becomes more spread out in contrast to the distribution of average annual rates of return. In other words, the lower tail of the distribution of outcomes becomes longer as the time horizon lengthens. Although the probability of an extremely bad outcome falls as T rises, the potential severity becomes worse.

Figure 1 shows the results.

The conclusion is that the probability of a shortfall is a flawed measure of risk because it ignores how severe the potential shortfall might be. Risk depends on the probability of a loss and on how large the loss might be. This generally is understood by individuals who routinely buy insurance against a variety of low-probability events such as their house burning down because of the severity of the consequences. The cost of shortfall insurance represents the insurance market's full assessment of both the probability and severity of possible losses.

Insurance against a shortfall in T years is effectively a put option. The put is of the European type, i.e., it can be exercised only at the expiration date, and it expires in T years. The put's exercise price is the insured value of the portfolio. If at the expiration date T years from now the portfolio's value exceeds its insured value, then the put expires worthless. If, however, there is a shortfall, then the put's payoff is equal to the shortfall.

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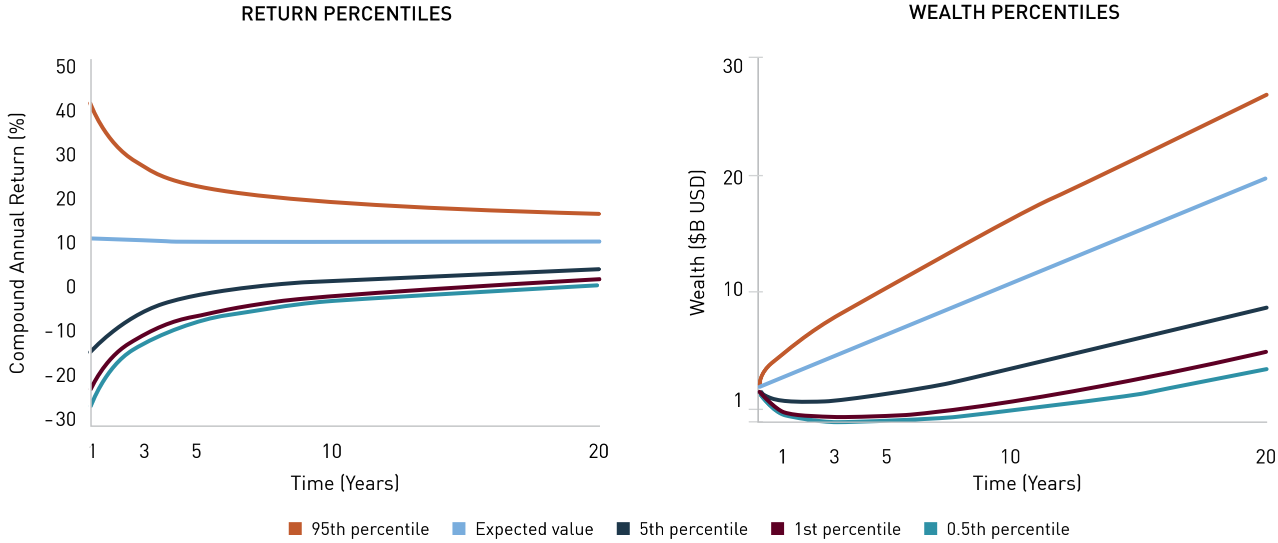
Figure 1

PROBABILITY DISTRIBUTIONS OF THE AVERAGE RATE OF RETURN VS. DISTRIBUTION OF FINAL WEALTH



Source: Bodie Academy of Financial Education

Let P be the cost of shortfall insurance per dollar invested. Then for each dollar insured against a shortfall, the total amount actually invested at the starting date is $1 + P. The exercise price of the put equals the price of the underlying stock portfolio compounded at the risk-free T-year interest rate.[6] Therefore the put-call parity theorem tells us that the price of the put equals the price of the corresponding call.[7]

To show that the value of the put increases with T, we could use any option-pricing model based on the condition that the financial markets do not allow anyone to earn risk-free arbitrage profits. Because it is so compact and so widely used in practice, we will use the Black–Scholes formula.[8] In our special case, the formula reduces to a relatively simple form. Moreover, with no loss of generality, we can express the price of the put as a fraction of the price of the stock:

$$\frac{P}{S} = N(d_1) - N(d_2)$$

$$d_1 = \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = \frac{-\sigma\sqrt{T}}{2}$$

where:

S = price of the stock

T = time to expiration of the option in years

σ = standard deviation of the annualized continuously compounded rate of return on the stock

$N(d)$ = the probability that a random draw from a standard normal distribution, is less than d.

Note that P/S is independent of the risk-free interest rate and the risk premium on stocks; it depends only on σ and T. Table 1 and figure 2 show the result of applying the formula to compute P/S assuming the annualized standard deviation of stock returns is 20 percent. The cost of the insurance rises with T, the term of the insurance. For a one-year term, the cost is 8 percent of the investment. For a 10-year term, it is 25 percent, and for a 50-year term it is 52 percent. As the term grows, the cost of the insurance approaches 100 percent of the investment.[9]

Some economists and other market observers have claimed that stock returns do not follow a random walk in the long run. Rather, they argue, the behavior of stock returns is best characterized as a mean-reverting process.

Some economists and other market observers have claimed that stock returns do not follow a random walk in the long run. Rather, they argue, the behavior of stock returns is best characterized as a mean-reverting process. It is mean reversion in stock returns, some say, that is the reason stocks are less risky for investors with a long time horizon. But figure 2 is valid for mean-reverting processes too. The reason is that arbitrage-based option pricing models, such as the Black–Scholes or binomial models, are valid regardless of the process for the mean. They are based on the law of one price and the condition of no-arbitrage profits. Investors who have different estimates

Table 1

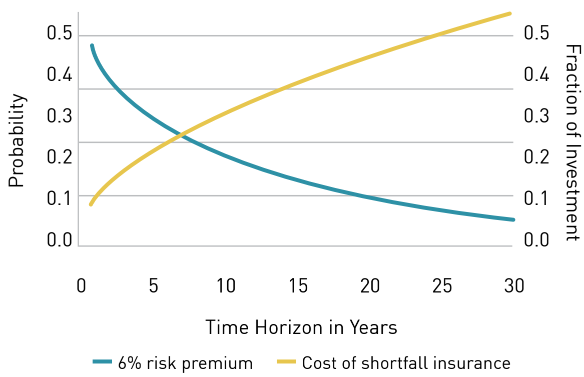
COST OF SHORTFALL INSURANCE AS A FUNCTION OF TIME HORIZON

Length of Time Horizon in Years	Cost of Insurance as Percentage of Investment
0	0
1	7.98
5	17.72
10	24.84
20	34.53
30	41.61
40	47.29
50	52.05

Note: The table was derived using the Black-Scholes formula with σ = 0.2 per year. The cost of the insurance is independent of the risk-free rate.

Figure 2

PROBABILITY OF A SHORTFALL VS. COST OF INSURANCE AS A FUNCTION OF TIME



Source: Bodie Academy of Financial Education

of the mean will agree on the price of the put as long as they agree about volatility.

Mean reversion is not sufficient to invalidate the relation depicted in figure 2. Stock prices would have to behave just like the price of a T-period zero-coupon bond that converges toward the bond's face value as the horizon date approaches. In other words, stocks would have to be indistinguishable from the risk-free asset for a T-period horizon.

It is not only in theory that the price of the shortfall put increases with T. The prices of actual puts traded on the exchanges follow this pattern, and an examination of the pricing offered by firms that sell puts over the counter will verify that their price schedule conforms to this pattern.

I have given estimates of the cost of insuring against a shortfall under the Black-Scholes model. But Black-Scholes assumes that the path of the stock price index is continuous. It is called a diffusion process. In reality stock prices will jump every so often causing volatility to jump significantly. Indeed, as Merton showed, the simple diffusion for the evolution of stock prices is reasonable when information arrives continuously in small increments of information per unit time (Merton 1976a, b). But sometimes information arrives in large chunks in an instant, i.e., in a very short span of time, so the sample path of information and therefore the sample path of stock price is not continuous but is from time to time discontinuous and jumps. In general, the jumps can be up or down. In specific situations, it will be in a single direction as for a devaluation of a weak currency.[10]

Merton's two papers (1976a, b) provide the quantification and mechanism for extracting separate estimates for the likelihood of a jump in similar fashion to implied volatility for the continuous component. Whether the horizon is one day or 200 years, the jump component matters even if the likely arrival time for a jump is once every 20 years.

FAULTY STATISTICAL INFERENCE

Another reason people fall into the trap of thinking that stocks are not risky in the long run is that they rely on faulty statistical inference. They are persuaded by the stock market history of the United States and the United Kingdom that, with virtual certainty, stocks will outperform bonds over long periods. There are at least three arguments against drawing such a conclusion from the historical data. The first is the small number of independent observations of long-period returns, the second is survivorship bias, and the third is that bonds in the past were not protected against inflation.

Regarding the time-series evidence for the long-run performance of equities, Samuelson (1994) writes:

> *We have data for, say, 150 years of bond and stock price quotations. That's about 2,000 monthly observations, 50,000 daily observations, and in some cases up to 400,000 hourly observations. It does not follow that we have 4 x 10^5 bits of information in favor of the proposition that over investment periods longer than ten years, the higher your equity exposure, the higher your cumulative return ... Overlapping moving averages are not each independent data points or degrees of freedom.*

With regard to survivorship bias, Brown et al. (1992) write:

> *Looking back over the history of the London or the New York stock markets can be extraordinarily comforting to an investor—equities appear to have provided a substantial premium over bonds, and markets appear to have recovered nicely after huge crashes ... Less comforting is the past history of other major markets: Russia, China, Germany, and Japan. Each of these markets has had one or more major interruptions that prevent their inclusion in long-term studies.*

In their empirical study designed to correct for survivorship bias, Jorion and Goetzmann (1999) write:

> *By putting together a variety of sources, we collected a database of capital appreciation indexes for 39 markets with histories going back as far back as the 1920s. Our results are striking. We find that the United States has by far the highest uninterrupted real rate of appreciation of all countries, at about 5 percent annually. For other countries, the median real appreciation rate is about 1.5 percent. The high return premium obtained for U.S. equities therefore appears to be the exception rather than the rule.*

JAPAN, A CAUTIONARY TALE

Advocates of the stocks-for-the-long-run hypothesis generally argue that in a well-developed capitalist system such as the United States, it is unimaginable that the real return over a long period would not be significantly positive. They contend that only in cases of war or other doomsday scenarios can one envision a negative outcome. Japanese experience provides a potent counter-example that easily refutes such armchair arguments. Since the end of World War II, the Japanese economy has flourished, yet its stock market has performed erratically. In the 1980s, Japan was the second-largest economy in the world and many experts were predicting it would overtake the United States by the end of the century.

Table 2 shows the value of the Nikkei 225 index from 1984 to 2020. It peaked in 1990 at 38,951; hit a low point 20 years later in 2009 at 7,909, and at year-end 2020 it was at 23,828. Thus after 31 years it is down 39 percent from its peak. This could happen in the United States or any other country.

Table 2

THE NIKKEI 225 STOCK PRICE INDEX—1984–2020

Japan: Second/Third Largest GDP in the World	Nikkei 225 Index 1984–2020
1984	10,071
1989	31,577
1990	**37,242** (high 38,951)
1991	23,271
1992	22,076
1993	17,038
1994	20,256
1995	18,647
1996	20,806
1997	18,308
1999	14,544
2000	19,537
2001	13,741
2002	10,027
2003	8,286
2004	10,785
2005	11,422
2006	16,929
2007	17,377
2008	13,518
2009	7,909
2010	10,212
2011	10,282
2012	8,789
2013	11,153
2014	14,414
2015	17,913
2016	16,060
2017	19,469
2018	21,182
2019	21,449
2020	23,828

MAKING PARTICIPANTS AWARE OF THE RISK OF EQUITIES IN THE LONG RUN

One way to make people aware of the long-run risk of equities would be to offer them competitively priced guarantees against a shortfall. Even if no one were to buy them, the prices of the guarantees would reflect the risk they were mitigating. Regulators could require that as a condition for a retirement investment or product to be considered a QDIA, the institutions offering them must offer as an option to guarantee a specified minimum benefit. Such consumer protection laws are common in the case of new cars and other durable consumer goods, and they ought to be applied to retirement-income products and investments. The principles of financial engineering can and should be used to design and produce such guarantees and manage them efficiently.

ACTUARIAL PRACTICE AND PENSION FINANCE

In a defined-benefit pension plan, the employer who sponsors the plan promises to pay retirement benefits based on a formula. The accruing benefits are backed by employer and employee contributions made to a fund that is invested and eventually produces the cash to make the promised payments as they come due. The safest investment strategy for both the plan sponsor and the plan beneficiaries is to invest in a portfolio of fixed income instruments that will produce the cash exactly when it is needed to pay the benefits. This asset-liability matching or liability-driven investing strategy is called immunization of the pension liability. When this procedure is followed, the cost of accruing benefits and the present value of the liability are computed using as the discount rate the interest rate on the fixed income portfolio that immunizes the liability.[11]

Bader and Gold (2003) provide a comprehensive critique of the actuarial model and advocate accepting the approach of financial economists. They suggest that public plans measure the liability using the rate on bonds regardless of how they choose to invest the assets in the fund.

But most state and local governments in the United States do not follow this procedure. According to the Urban Institute, states and localities follow pension accounting standards set by the GASB:

> *The standards require pension plans to retain actuaries to project future assets and liabilities based on demographic and economic assumptions. Actuaries then calculate the employer contributions necessary to cover liabilities incurred by current employees plus any amounts needed to address past unfunded liabilities.*[12]

In practice, however, it appears that most state and local government pension plans have a discount rate similar to the expected return of the plan's portfolio, which is generally a mix of stocks, bonds, and other asset classes.[13] Plan fiduciaries, with the support of actuarial consultants, argue that they have a long time horizon, and in the long run a diversified portfolio of stocks will earn an assumed 7-percent rate of return with very high probability, and it is virtually certain that it will earn a rate of return at least as great as the rate of interest on bonds.

So, they invest their contributions to the pension fund in a portfolio that is mostly in stocks. They then calculate the cost of accruing pension benefits to employees by assuming that the funds invested to pay those benefits will earn a rate of return equal to the roughly 7 percent per year earned in the past. That rate is then used by actuaries to compute the present value of the pension liabilities on the state or local government's pension plan balance sheet instead of the current interest rate on bonds, which is now roughly 2 percent.[14]

Thus, the pension accounting procedure for public plans amounts to acting as if there is an arbitrage opportunity allowing these plans to "earn" roughly 5 percent on each dollar of pension liabilities they have. Even though it is just wishful thinking, many state and local governments have treated it as a genuine arbitrage opportunity. They have issued "pension obligation bonds" to fund their unfunded pension liabilities. These are bonds issued by the local government at a fixed rate of interest, and the cash raised is invested in a portfolio of stocks.[15] In reality, this set of transactions amounts to a highly levered investment in the stock market—a fake arbitrage position.

Consequently, most state and local government pension plans in the United States today are massively underfunded (Novy-Marx and Rauh 2011). In 2017 the total pension liability under GASB 67 standards for all state and local funds was $5.242 trillion, which is covered by $3.859 trillion in assets, which implies an unfunded liability of $1.733 trillion and a funding ratio of 73.6 percent. These figures assume a liability-weighted average discount rate of 7.36 percent. However, under market-value standards, the total accumulated benefit obligation liability is $8.012 trillion. Compared to the $3.859 trillion in assets, this implies a true unfunded market value liability of $4.153 trillion and a funding ratio of only 48.2 percent. These figures assume an average liability-weighted Treasury discount rate of 2.77 percent.[16]

To remedy this flaw in measurement and asset allocation of public pension plans, the required policy is clear. The GASB's accounting standards must be changed to require that the liability be evaluated using a discount rate that reflects the interest rate on a replicating portfolio of bonds. This could lead to a change in asset allocation strategy, as well as other changes.

CONCLUSION

It is widely believed that although stocks are risky in the short run, in the long run they are sure to outperform risk-free investments such as government bonds. This is a dangerous fallacy. It implies that the stock and bond markets provide unlimited arbitrage opportunities. In this paper, I explored some of the consequences of the fallacy with respect to (1) the regulations governing employer-sponsored retirement plans that enroll participants in QDIAs and (2) the measurement and funding of pension promises by state and local governments.

I measured the risk of stocks as the market price of insurance against earning less than the risk-free interest rate. Such an insurance policy is equivalent to a European put option with strike price equal to the forward price of the underlying stock index. The price of such a shortfall put increases—not decreases—with the time to expiration. Next, I explored the harmful effects that the fallacy has had in the past and continues to have in the present. These harmful effects are:

- In defined contribution plans, the U.S. Department of Labor has set rules for QDIAs that discourage stable value funds and encourage investment in stocks. The result is that uninformed plan participants who are automatically enrolled in QDIAs are exposed to more market risk than they are aware of.
- In valuing pension liabilities in state and local government defined benefit pension plans, actuaries use discount rates that are too high. The result is underfunding of those liabilities, which can and does lead to bankruptcy of those plans.[17]

Finally, the paper recommends policy measures to counteract the harmful effects of the fallacy. The first is for regulators to require that as a condition for a retirement product, investment, or service to be considered a QDIA, the institutions offering them also must offer an option that would provide a guarantee of a specified minimum benefit. Such consumer protection laws and regulations are common in the case of new cars and other durable consumer goods, and they ought to be applied to QDIAs. The principles of financial engineering can and should be used to design and produce such guarantees and manage them efficiently.

The second is to convince the GASB to change its standards of practice regarding the valuation of pension liabilities and the investment of pension assets. There are, of course, hurdles to be faced and critics to address with this change. First, in its deliberations surrounding its new pension accounting standards, the GASB considered and rejected requiring the use of a risk-free rate to public pension plans. Two, investing exclusively in fixed income assets could result in drastically lower investment returns. And three, critics of this recommendation suggest that using risk-free rates to discount liabilities generates other risks, such as overcharging current taxpayers and undercharging future taxpayers when rates are low, and the reverse when rates are high. These hurdles and critics would, of course, need to be addressed.

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ENDNOTES

1. Established in 1984, the Governmental Accounting Standards Board (GASB) is the independent, private-sector organization based in Norwalk, Connecticut, that establishes accounting and financial reporting standards for U.S. state and local governments that follow Generally Accepted Accounting Principles (GAAP). https://www.gasb.org/home.
2. See GASB's New Pension Standards: Setting the Record Straight: https://www.gasb.org/cs/ContentServer?cid=1176160432178&d=&pagename=GASB%2FPage%2FGASBSectionPage and GASB 67 & 68 and the Changes Impacting Reporting and the Auditing of Pension Data: https://www.in.gov/sboa/files/McGladreyGASB_67_68_SBOA_Presentation.pdf.
3. A case in point was the bankruptcy of the city of Detroit in 2013–2014.
4. See https://www.morningstar.com/start-investing/classroom.
5. Samuelson was the first prominent thought leader to disprove the conventional wisdom about stocks in the long run. His 1969 paper demonstrated that in the standard life-cycle model, the fraction of one's assets allocated to stocks would not vary with the length of the time horizon.
6. Another way to state this is that the exercise price of the put equals the forward price of the underlying stock.
7. The put-call parity theorem for European options states:
$$P + S = C + E\,e^{-rT}$$
where P is the price of the put, S is the price of the underlying stock, C is the price of the corresponding call, E is the exercise price, and r is the risk-free interest rate. In our case: $E = Se^{rT}$.
By substituting into the put-call parity relation we get: $P = C$.
8. The reference here is to the option-pricing theory originally developed by Black and Scholes (1973) and Merton (1973). There is an extensive literature on using option-pricing models to estimate the value of financial guarantees. For a comprehensive list of references, see Merton and Bodie (1992).
9. Note that P is not equal to the expected value of the shortfall. However, if risk-neutral probabilities are substituted for actual probabilities, then one arrives at P.
10. This was called the "peso problem" by Krasker (1980).
11. Bader and Gold (2003) provide a comprehensive critique of the actuarial model and advocate accepting the approach of financial economists.
12. See Urban Institute, State and Local Government Pensions, https://www.urban.org/policy-centers/cross-center-initiatives/state-and-local-finance-initiative/projects/state-and-local-backgrounders/state-and-local-government-pensions.
13. Based on the latest information from the National Association of State Retirement Administrators (NASRA) Public Fund Survey, the average public pension fund asset allocation is as follows: public equities: 47.1 percent; fixed income: 24.0 percent; real estate: 7.4 percent; alternative investments: 19.4 percent; cash and other: 2.0 percent. The median discount rate for state pension plans was 7 percent as of July 2021. This NASRA brief discusses how investment return assumptions are established and evaluated: https://www.nasra.org/content.asp?contentid=120.
14. The practice of using the expected rate of return on pension assets to discount the liabilities is recommended under the GASB Statement 67. https://www.gasb.org/jsp/GASB/Pronouncement_C/GASBSummaryPage&cid=1176160219444.
15. In 1986, the federal government ruled that in such transactions state and local governments lose the tax exemption on their bonds.
16. See Stanford, Pension Liability Map, http://web.stanford.edu/~rauh/faq/charts.html.
17. Pension bankruptcy led to the bankruptcy of the city of Detroit in 2014. Detroit filed for bankruptcy on July 18, 2013, and emerged from it on December 10, 2014.

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