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ABSTRACT

In the process of evaluating various retirement income strategies, it appears that, for investors not endowed with substantial wealth relative to consumption demands, sequence of returns risk is operative throughout retirement. We explore, in an asset-liability modeling context, the reasons why sequence risk exists throughout the planning horizon and why it can be particularly acute at the end of an investor's life span. Given the nature of this risk, prudent asset management benefits from developing appropriate risk metrics, and from implementing credible monitoring, evaluation, and communication procedures. Two case studies focus on sequence of returns risk. They present risk metrics designed to answer the following questions: (1) is the investor's retirement income strategy feasible; (2) if yes, is it sustainable; and (3) does it allow sufficient flexibility to provide security in the face of financial shocks? The risk metrics employ information derived from both investment simulation models and actuarial calculations.[1]

MONITORING THE CONTINUED FEASIBILITY OF CLIENT OBJECTIVES

Asset management encompasses monitoring and evaluating initial investment decisions. Monitoring a retirement income portfolio incorporates assessments about current resources and the likely future consequences of asset management decisions—stay the course or change strategy. Given the potentially debilitating effect on a newly retired investor's portfolio of an unfavorable sequence of investment returns, portfolio monitoring and evaluation now are viewed as critical components of the investment advisor's service set.

Although investors may be interested in evaluating past performance, many want to focus on what should be done today in order to secure a successful economic future. More technically, how does the investor assess probabilities about future outcomes given the current information set? Are these outcomes acceptable? What asset management elections should the investor consider now?

Technically, monitoring and evaluation is a process of forming posterior probability estimates or conditional probability estimates. We borrow vocabulary from the ideas originally presented by Reverend Thomas Bayes.[2] Initial portfolio design and asset management decisions reflect return and risk preferences based on the investor's *a priori* probability distribution.[3] Given the sequence of returns realized since the original investment decisions, what is the likely financial outcome for a client's retirement plan? Once the investor understands how the portfolio arrived at its current state, the issue becomes how to update beliefs when faced with new evidence.[4] From the perspective of an advisor, the task is to help a client (1) understand events and learn from them as time unfolds and (2) use the updated information to reassess preferences and, perhaps, modify behaviors. If new information is favorable, the investor may have an opportunity to increase consumption, gifting, or bequest levels. If new information is unfavorable, the investor may have an option to maintain or decrease consumption, gifting, or bequest levels. Also included in the decision set is the option to change investment management strategy. For example, if faced with changes in economic circumstances, is it prudent to initiate or delay an option to implement "flooring?" When is it appropriate to reset the stock/bond allocation for a buy-and-hold investing strategy? When is it prudent to acquire an annuity, a bond ladder, or other similar asset-protection strategies or products? Or, more particularly, to what extent has a return sequence jeopardized or enhanced the investor's financial future?

Generally, investors do not think in probabilistic terms. Rather, they ask: Are my goals achievable? Am I gaining or losing ground financially? These are the topics of interest and the advisor's challenge is to translate quantitative insights into qualitative, i.e., understandable, vocabulary.

THE ACTUARIAL COVERAGE RATIO: A USEFUL MONITORING METRIC

The initial challenge is to gauge the portfolio's current economic condition relative to the client's future economic goals. Have the nature and scope of assets and liabilities changed over time, and do goals remain feasible in light of current resources? One method of addressing the feasibility issue is to verify that the portfolio's current fair market value equals or exceeds the value of the liabilities it is asked to discharge. When liabilities are defined as the need to provide lifetime cash flows, the feasibility condition is satisfied whenever the portfolio maintains an economic surplus in the face of changes in its asset and liability values. Operationally, this requires updating, based on current information, the value of a metric that we term "actuarial coverage ratio" (ACR). The question of interest is whether the investor's retirement income objectives continue to remain feasible. Feasibility is, in some important respects, a function of the investment, inflation, and liability costs realized up to the current date—sequence risk. A goals-based monitoring program evaluates both the magnitude and direction of realizations in terms of client-specific objectives.

The ACR is a more stable tracking metric than portfolio value because it encompasses both retirement financial resources and liabilities.[5] The ACR, as presented in this discussion, is the ratio of the portfolio's current market value to the current estimated cost of an actuarially fair annuity—decremented by an assumed 15-percent "load" factor—paying the target amount of lifetime income.[6] Both portfolio market value and annuity cost (fair price + estimated load) are current observables and do not rely on a simulation engine's best guess.[7] The annuity pricing factor is the sum of periodic cash flows, where each cash flow is adjusted for the probability of remaining alive to receive it and discounted by the applicable yield curve. The cost of a lifetime unit of income (nominal or inflation-adjusted) changes with age, with changes in systematic (i.e., population) longevity risk, and with the discount rate. Actuarially fair annuities do not exist in the marketplace. Depending on (1) the investor's age and gender and (2) the insurance company's sales and administrative expenses, reserving requirements, and profit objectives, investors should expect to multiply the actuarially fair cost by a substantial load factor.[8] Stated otherwise, should an investor wish to exercise the option to trade financial wealth for an equivalent-value lifetime income stream, he must expect to pay a substantial cost. An ACR of 1 indicates that current wealth (numerator) exactly equals the estimated current cost of an annuity contract (denominator).[9]

Both rationalist and behaviorist commentators recognize that the risk aversion function may change across the wealth domain. As the ACR approaches a value of 1, reactions and solutions vary. Advisors, subscribing to the belief that it is optimal for investors to employ asset management strategies consistent with constant relative risk aversion, may recommend a stay-the-course response while advisors assuming that investors exhibit decreasing absolute risk aversion may suggest risk reduction in periods following substantial decline in portfolio value.

The ACR metric is variable if, for no other reason, the yield curve discount factor is not static. Realistically, the yield curve changes constantly and, even small changes in the discount rate may generate large changes in ACR value over the relevant planning horizon. This fact is of importance when considering the interrelationship between ACR values and retirement feasibility in the face of sequence of returns risk.

Feasibility evaluation, when using an actuarial benchmark, incorporates the following:

- Distribution and sequence of future investment returns
- Distribution and sequence of inflation realizations
- Distribution and sequence of retirement consumption (basic or aspirational standard of living goal)
- Distribution of future annuity costs—sequence of yield curve changes
- Health state differential (bad to worse, bad to good, etc.)
- Force and effect of mortality

Relationships among ACR factors are nonlinear and feasibility or solvency tests cannot be calculated on the back of an envelope. The advisor monitors and assesses these variables within a high-dimensional context.

This is a "how should I plan" rather than a "what should I own" approach. As time unfolds and new information arrives, investor preferences may change due to changes in economic circumstances (ability to accept risk) or in personal preferences (willingness to accept risk).[10] Ideally, there exists a credible method for taking such developments into account, understanding their economic impacts, and formulating prudent and suitable responses. An investor cannot predict how personal preferences or circumstances may change, but adopting a retirement-as-a-risky-option perspective allows him to overcome decision paralysis: "This should be particularly important in a situation where the agent must decide today which options he wants to leave open in the future."[11]

The ACR provides an important metric for reframing discussions of risks and rewards. A low ratio value suggests the need to consider corrective actions such as a spending reduction, a delay in a lifetime gifting schedule, implementation of a reverse mortgage, or some combination of these and other planning options. A high ratio value suggests the need to revisit spending and gifting or bequest objectives to ensure optimal balance according to the investor's preference weighting (utility).

ACTUARIAL COVERAGE RATIO, LONGEVITY RISK, AND PORTFOLIO VALUE

Retirement income risk modeling, from the perspective of ACR analysis, yields interesting insights into longevity risk. We consider longevity risk to be a type of "tail risk." Given the distribution of life spans, it is inevitable that some investors will survive well past their life expectancy. Some long-lived investors may incur the risk of portfolio depletion—outliving their assets.[12] A traditional characterization of longevity risk is the joint conditional probability of (1) living beyond life expectancy and (2) realizing investment returns insufficient to sustain the required lifetime target income. Consumer surveys suggest that retired investors worry about declines in portfolio value because they do not want to spend their final years unable to support an acceptable standard of living. They may express this concern in terms of not wanting to be an economic burden on other family members, not wanting to apply for public or charitable assistance, and, ultimately, not wanting to sink into poverty.

The investor must be wary of longevity risk as age increases. Each year represents a greater proportion of remaining life expectancy and each year pushes life expectancy further into the future. The longer you live, the longer you're expected to live. As time in retirement passes, investors place greater and greater emphasis on the dollar value of the portfolio if:

- The portfolio is on a downward dollar-value trajectory
- There is little flexibility in spending because current spending represents a greater proportion of portfolio value[13]

Intuitively, many investors, lacking substantial wealth, realize that they are in a race between death and depletion. The stress meter can redline during periods of portfolio decline.

As the retired investor spends a greater and greater proportion of the remaining portfolio to sustain the target standard of living, will outcome volatility increase? How does the investor know if he or she is winning the "race?" What are the sources of outcome volatility for the late-in-life investor? An analysis should consider the following:

- Unfavorable inflation realizations
- Unfavorable investment returns
- Increases in liability values
- Changes in longevity risk

These are not uncorrelated variables. In an asset-liability management context, the deleterious effects of asset value decline due to higher interest rates may, in part, be offset by a correspondingly higher discount rate for valuation of liabilities.[14] Changes in longevity risk are especially interesting. Obviously, longevity risk may change with health status. A cure for a disease, although having a beneficial impact on the general population, may have a detrimental economic impact on some retired investors if limited resources must be stretched further to accommodate longer expected life spans. However, there is another component to longevity risk.

Actuarial data indicate, for example, that life expectancy for a white-collar, high-income, male retiree in current good health at age sixty-five is approximately age eighty-eight. However, life expectancy at age eighty-eight for the population of surviving males is age ninety-three. This means, absent any change in systematic population mortality risk, the population of age sixty-five male investors fortunate enough to live to age eighty-eight will have a revised expectancy of surviving to age ninety-three. That is to say, half of the age eighty-eight population can expect to be alive beyond age ninety-three. But no one's giving the portfolio an extra five year's life expectancy.

If the initial age sixty-five investment planning contemplated the interactions of time, inflation, and investment returns to arrive at credible risk metrics and retirement income projections, by age eighty-eight the original numbers may be all wrong. Critical variables change merely with the passage of time.

SEQUENCE OF RETURNS RISK

As one ages, life expectancy increases; however, the number of years of expected remaining life decreases. At age sixty, an investor may have a thirty-year life expectancy (age ninety); at age eighty-nine, the investor, if surviving, may have a four-year life expectancy (increase to age ninety-three). The number of years of expected remaining life decreases from thirty years at age sixty to four years at age eighty-nine.[15]

What is "sequence of returns risk" and what does the above observation have to do with this risk? We first define and illustrate sequence of returns risk; we then connect it to longevity risk to illustrate a complex, and not commonly understood, interrelationship.

Mathematically, sequence of returns risk is usually explained in terms of path dependency. Here is a simplified example:

We know (because multiplication is commutative) that $3 \times 2 \times 1 = 6$ and that $1 \times 2 \times 3 = 6$. The order of the numbers ("returns") does not matter. This principle holds for any compound return series in which there are no cash flows. Consider, however, what happens when we introduce cash-flow requirements to the series. In this case, the investor withdraws one-half unit of value from the portfolio each period.

Period One: $1 \times 3 = 3 - ½ = 2½$

Period Two: $2½ \times 2 = 5 - ½ = 4½$

Period Three: $4½ \times 1 = 4½ - ½ = 4$ units of ending wealth.

However,

Period One: $1 \times 1 = 1 - ½ = ½$

Period Two: $½ \times 2 = 1 - ½ = ½$

Period Three: $½ \times 3 = 1½ - ½ = 1$ unit of ending wealth.

In the presence of withdrawals, the return order matters. An average return target is no longer a legitimate risk metric in the presence of portfolio distributions and return variance. Over time, when faced with the presence of cash flows, one can be exactly on track with respect to a portfolio's expected average return but wildly off target with respect to the portfolio's actual dollar value. It is the dollar value, however, that must support financial objectives.

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A common method for illustrating this risk uses the sequence of returns from the S&P 500 stock index during 1975 through 2009. If a portfolio, absent fees, taxes, trading costs, and other frictions, is completely and continuously invested in the S&P 500 Index, and if there are no contributions to or withdrawals from the portfolio during this period, the order of periodic returns has no effect on terminal wealth. Making an initial investment of $1,000 generates the same ending wealth if you reverse the historical return pattern, preserve the historical pattern, or scramble the returns. However, if you take $50 per year from the portfolio under the historically realized return pattern, your ending wealth is positive; if you reverse the return order,[16] you end up broke; and, if you scramble the order, God knows what terminal wealth the return series generates. Final results are path dependent.

Many commentators advance the proposition that investor exposure to sequence of returns risk is greatest at the beginning of retirement. They correctly point out that when declines occur in the early retirement years, negative returns have an outsized effect. Early-in-retirement declines operate on substantial wealth, and the resulting loss of initial wealth compounds for many years. If late-in-life wealth diminishes, declines operate on smaller dollar-valued portfolios. Therefore, some argue, the impact on the investor's financial security is less severe because the dollar wealth is less.[17]

Results obtained from our risk model[18] suggest that sequence of returns risk is operative throughout retirement. Irrespective of the dollar value of a portfolio, withdrawals at a dollar amount sufficient to sustain the investor's standard of living must continue. Withdrawals by some late-in-life investors may constitute a significant proportion of remaining wealth. Such investors are highly vulnerable to sequence of returns risk because they have limited risk capacity.

CASE STUDY 1: SEQUENCE OF RETURNS RISK FOR THE LATE-IN-LIFE INVESTOR

In the following case study, two things occur simultaneously:

1. The investor withdraws larger and larger portions from a smaller and smaller portfolio each year.
2. Each year of life pushes the applicable planning horizon further into the future.

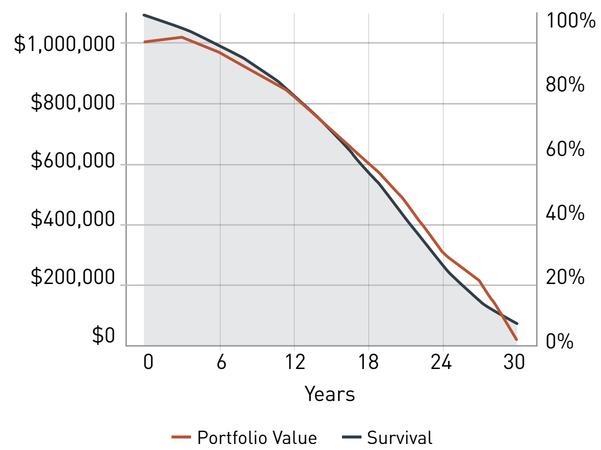
It is the interaction of these two factors that creates late-in-life vulnerability to sequence of returns risk.

A healthy seventy-year-old woman wishes to withdraw $4,600 per month on an inflation-adjusted basis from a portfolio allocated to only two investment positions:

- 65 percent to the S&P 500 Index
- 35 percent to the U.S. Aggregate Bond Index

Figure 1

AGE 70: SURVIVAL RATE AND EXPECTED PORTFOLIO VALUE



The risk model generating the portfolio operates as specified in appendix A. The investor lacks gifting and bequest motives, but she seeks lifetime budgetary certainty. A risk model simulating returns by means of a Markov transition matrix with the initial economic state selected randomly from either a bull market or a bear market suggests that her portfolio value "in expectation" (fiftieth percentile of results) should closely track her longevity, with the result that, even if she lives to age 100, she will consistently achieve her target income goal (see figure 1).

Fast forward fifteen years. The investor takes comfort from several fortunate outcomes:

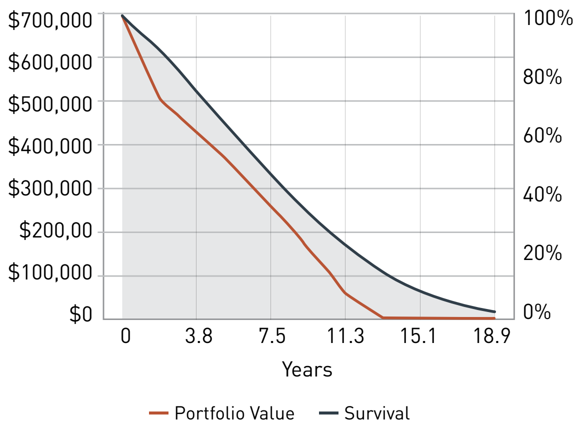
1. Her health has remained good despite the fact that she is now age eighty-five.
2. Her portfolio has successfully weathered inflation and return vicissitudes and has generated the expected dollar value at the beginning of year 15, namely, $693,500.

Is she now less vulnerable to sequence of returns risk?

The answer may be yes, if the survival expectation curve remains static. However, as the investor ages, the survival probability curve shifts further to the right.[19] As the investor grows older, in some ways she becomes more susceptible to sequence of returns risk because a bad investment return combined with a larger proportional share of wealth withdrawn for consumption can cause immediate problems. On the other hand, if her proportionate consumption is large relative to the returns, then, in another sense, one may argue that her consumption of principal—not investment returns—is the dominating risk factor. However, as age increases in this case study, the impact of market declines becomes more pronounced because consumption, geared to deplete principal as the ACR value becomes less favorable, takes an ever-larger bite.

Figure 2

AGE 85: SURVIVAL RATE AND EXPECTED PORTFOLIO VALUE



Consider figure 2, which begins with the investor's financial position at age eighty-five. At this time, the investor encounters a bear market. By year 12 (year 27 in terms of the age-seventy analysis in figure 1) she is out of money. However, she now has a longer survival expectation—approximately 20 percent of the initial population of 10,000 sample investors will, at age eighty-five, survive to age ninety-seven.

Whenever a limited amount of financial resources must sustain lifetime income, portfolio monitoring and evaluation are central to gauging investor retirement income security. Given that the investor's expected longevity increases as she ages, all else being equal she remains vulnerable to downturns in portfolio value. But the extent of vulnerability is measured by the value of the ACR. An ACR slightly above 100 percent indicates financial vulnerability.[20] By contrast, if the older retiree's ACR exhibits a high value, the investor is less vulnerable to sequence of returns risk.

One implication is that, for retired investors of any age, the ACR is a key evaluation metric. An investor age sixty-two beginning retirement with an ACR equal to 110 percent is exactly as vulnerable to sequence of returns risk as an investor age ninety-two with the same ACR value.[21] Advisors should redefine sequence of returns risk. It is a function of resources, longevity expectations, and consumption demands. Retirement, when faced with ACR values close to 1, is a risky project at any age.

CASE STUDY 2: A DYNAMIC LOOK AT SEQUENCE OF RETURNS RISK

Case Study 2 considers a well-diversified retirement income portfolio. Many retired investors define income security as having the amount of money they planned on having at the time they planned to have it. Fixed wealth, however, often demands fluctuating income,[22] and fixed income often demands fluctuating wealth.[23] If wealth is an asset that provides money to support financial goals, then the savvy investor is at least as concerned with the safety and sustainability of the goals as with transitory price levels of various global financial markets. As Robert Merton (2014) wrote: "… an investment that is risk-free from an asset value standpoint may be very risky in income terms." However, many investors focus primarily on asset value. A disconnect in thinking occurs whenever an investor confuses stability in asset value with stability and sustainability in cash flows. The market watchers and prognosticators seldom speak in terms of a portfolio's ability to provide resources, over time, to pay the bills. Their vocabulary tends to trap listeners into a short-term dynamic based on fear and greed; their exhortations encourage buy-sell strategies that, more often than not, enrich only the brokers. If the goal is lifetime income, the primary risks are income insufficiency and the failure to achieve income sustainability (shortfall risk metrics), not portfolio value fluctuations.

In Case Study 2, an advisor meets an investor who passes initial tests for retirement feasibility (current ACR coverage ratio equals 138 percent, see table 1). She is age seventy-four, in good health, and has implemented a portfolio allocated 50 percent to stocks and 50 percent to fixed income securities.[24] The portfolio's current value is $5 million and its lifetime periodic income target is $20,000 per month adjusted for inflation.

The advisor congratulates the investor and remarks that the ACR value indicates that she can comfortably embark on her preferred retirement income strategy. She has the expectation that the portfolio can adequately fund anticipated future spending. In a static world, the investor can relax because values of key variables will not change. Under conditions of uncertainty, however, this may not be fully possible. The ACR reflects the interaction of critical variables (investment, inflation, health, interest rate, and liabilities) each of which may manifest changing parameter values as "realizations" emerge over time. Rather than average values (point estimates), the investor faces a multi-dimensional distribution of possible outcomes (range estimates). Granted that analysis confirms the goals are initially feasible, nevertheless, it is a point-in-time estimate (today) that bears close monitoring (tomorrow). The sequence of returns risk is fully operative in this context.

The investor says that she understands that the future is not set in stone, but she is unsure about how ACR values can change from favorable to unfavorable. If she has enough money to retire today, what is the possibility of waking up some morning to discover that this is no longer true? The advisor suspects that it is time to add a dynamic time dimension to the retirement income feasibility analysis.

The advisor states that he will present, in tabular form, a distribution (range) of future results from an asset-liability

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Table 1

DISTRIBUTION (RANGE) OF FUTURE RESULTS FROM AN ASSET-LIABILITY MANAGEMENT PERSPECTIVE

	Years From Today										
	0.0Y	2.8Y	5.8Y	8.7Y	11.6Y	14.5Y	17.4Y	20.3Y	23.2Y	26.2Y	29.1Y
95%	138%	261%	327%	415%	557%	746%	1001%	1386%	1914%	2184%	3790%
90%	138%	228%	279%	350%	453%	596%	782%	1051%	1342%	1981%	2684%
80%	138%	190%	227%	275%	339%	431%	550%	713%	857%	1215%	1575%
70%	138%	167%	194%	231%	277%	343%	422%	528%	603%	742%	973%
60%	138%	147%	170%	197%	231%	274%	322%	384%	395%	473%	687%
50%	138%	131%	147%	168%	192%	216%	244%	268%	250%	253%	250%
40%	138%	116%	126%	140%	155%	169%	171%	169%	112%	60%	15%
30%	138%	100%	107%	114%	123%	122%	109%	82%	2%	0%	0%
20%	138%	83%	86%	87%	85%	75%	45%	0%	0%	0%	0%
10%	138%	62%	61%	57%	45%	22%	0%	0%	0%	0%	0%
5%	138%	47%	44%	38%	20%	0%	0%	0%	0%	0%	0%

Green = surplus; Yellow = awareness; Orange = caution; Red = danger

management perspective.[25] For the spending strategy under consideration ($20,000 per month lifetime, inflation-adjusted, portfolio withdrawal), the distribution's range indicates the likelihood of a favorable ACR value continuing for the long term. He explains that an ACR value less than 100 percent suggests that reducing future spending may be prudent and an ACR value sufficiently greater than 100 percent suggests that the investor may have an option to increase consumption. A low value (<100 percent) indicates that the investor has a negative surplus in an asset-liability management context, and that she has entered into a danger zone.

The advisor explains that the table is a heat map. The green-shaded areas are regions of "surplus." The ACR value of 100 percent is the feasibility boundary. A value below 100 percent indicates that resources are potentially inadequate with respect to the future monetary demands placed upon them. As the investor approaches the feasibility boundary from above, she enters into the yellow-shaded areas, which are regions of "awareness." If the ratio value moves closer to the lower bound, the color changes to orange, which signifies a region of "caution." Finally, if the portfolio penetrates the feasibility boundary, the red-shaded areas are regions of "danger."[26] When the investor remarks that the table exhibits a heavy dose of "danger," the advisor cautions her not to jump to a hasty conclusion—things are not as perilous as they may first seem. He asks for permission to explain further the table's informational content.

The entire table represents a distribution of possible ACR values. After stating that the table shows a distribution of ratios, the advisor pauses for a moment. Failing to see any sign that the lightbulb over the investor's head has clicked on, the advisor admits that the concept of a "distribution of ratio values" is tough to comprehend. The advisor takes a sheet of paper and writes some fractions:

$$\frac{\textit{Assets}}{\textit{Liabilities}} = \frac{\textit{Wealth}}{\textit{Consumption}} = \frac{\textit{Value of Portfolio}}{\textit{Sum, in current dollars, of future spending assuming you're alive to buy things}}$$

By definition, a fraction has a numerator and a denominator. This makes it a ratio—such as miles per hour (distance ÷ time) or miles per gallon (distance ÷ number of gallons). The concept of ACR is complex because both the numerator and the denominator have multiple, and continuously moving, parts. The value of the portfolio in the numerator is an ever-changing number dependent on the sequence of random variables of inflation and investment returns—sequence of returns risk.[27] The discounted sum of mortality-adjusted spending in the denominator constantly changes as a function of time (age) and of liability values (changes in the discount rate used to value liabilities or, in the nature and scope of liabilities themselves, sequence of liability values risk). In Case Study 2, the denominator is the actuarial cost, at a particular moment in time, for a guaranteed lifetime income. Given so many moving parts (a person's financial situation never remains static because, all else being equal, it changes merely with the passage of time), it is no wonder that ratio values are variable. The simple answer is "things change," and, this is why retirement is a risky project. There is uncertainty in the numerator (assets) and in the denominator (liabilities), and both elements contribute to overall retirement risk.

Returning to table 1, the far-left column divides the ACR distribution into percentiles ranging from "worst case" (fifth percentile) to "best case" (ninety-fifth percentile). Values in red indicate a negative surplus—an ACR less than 100 percent. The horizontal axis across the top records points in time for the evolution of ACR values over the applicable planning horizon (to approximately age 103). At time zero (0.0Y = today), the investor's ratio value (current portfolio value ÷ discounted value of future income targets calculated under the annuity pricing principle) equals 138 percent. Her retirement spending strategy is feasible; and, because today's ACR is based on current observables (market value of portfolio and actuarially determined cost of lifetime income), there is no guesswork—the ACR value is 138 percent at every percentile in the distribution of possible results. Her retirement is feasible.

A key to understanding the table, according to the advisor, lies in the wording of the fraction:

$$\frac{\textit{Value of Portfolio}}{\textit{Sum, in current dollars, of future spending assuming you're alive to buy things}}$$

Here is the advisor's explanation in non-mathematical terms, "We know that the portfolio's value—the numerator—will change over time. However, the actual length of time for portfolio withdrawals is measured by an uncertain life span. Thus the key phrase is 'assuming you're alive to buy things.'" Or, "if you're alive in the future, what is the likelihood your portfolio will run out of money (hit an ACR value of 0 percent)?"

We know that the portfolio's value—the numerator—will change over time. However, the actual length of time for portfolio withdrawals is measured by an uncertain life span.

The initial sample consists of 10,000 simulated trials where each trial charts the inflation, mortality, and investment return paths for a seventy-four-year-old, healthy, female investor. At time zero, the investor is alive in each trial. Table 1 does not record an ACR value of 0 percent until time 14.5 years. At that time, for the investors who still remain alive to buy things, approximately 5 percent will have fully depleted portfolio wealth assuming no mid-course spending adjustments. At time 20.3 years (investor age ≈ ninety-four), of the remaining simulated trials, approximately 20 percent of those who are still living will have run out of money assuming no mid-course spending adjustments. Finally, at time period 29.1 years (investor age ≈ 103), of the few remaining simulation trials, approximately 30 percent of those still living will have run out of money assuming no mid-course spending adjustments.

The advisor further clarifies the informational content embedded in table 1. The state of "alive-and-broke," represented by an ACR value of zero, only occurs at the intersection of three events: (1) the investor is fortunate enough to enjoy a long life span, (2) investment or inflation realizations are particularly unfavorable during the planning horizon and, perhaps most significant, (3) no one notices the unfolding economic danger and recommends corrective actions prior to wealth depletion. The heavy dose of red that initially alarmed the investor reflects conditional, rather than absolute, probabilities. In order to enter into the region of danger, the investor has to remain alive, suffer poor investment results, and remain unaware of her economic plight (or, remain unwilling or unable to revise her spending policy or investment strategy).

The investor remarks:

> *When I initially looked at the* table, *I thought that it raised the specter of an approximately 30*-percent *chance of running out of money if I elect a $20,000 monthly, inflation-adjusted income. This is an unacceptable* risk. *However, if I understand the concept of conditional probability, I see that it suggests that approximately* 30 percent *of the very few sample investors who remain alive at age 103, and who continue the full monthly distribution from the investment portfolio, experience economic difficulty.*

Consider table 1, once again. If you trace the yellow and orange caution zones throughout the planning horizon, there appears a pattern that we term "the danger smile." The warning or danger zone areas dip in the middle and rise at the beginning and end. In this case—it is difficult to generalize—there is clearly greater likelihood of a negative outcome at the beginning and the end of retirement. This pattern can be attributed to the outsized deleterious impacts of initial bad investment returns, the effect of which dissipates as multiple bad return periods become unlikely and the upward trend in expected return takes hold. However, at advanced ages, there is also greater sensitivity to sequence of returns risk because a few additional years of life make a big difference to ACR values at that time. If we were pressed to identify the ages for which the sequence of returns risk is greatest, we would surmise: (1) the very beginning of retirement for most investors with modest ACR values and (2) the end of retirement for longer-lived investors.

The Case Study 2 analysis assumes an investor who prefers budgetary certainty ($20,000 per month constant-dollar withdrawal from the portfolio). Do conclusions change for other withdrawal strategies? Is sequence of returns risk only an artifact of a particular retirement consumption profile?

Appendix E illustrates heat-map tables for two additional withdrawal strategies with all other aspects of the fact pattern held constant:

A hybrid withdrawal strategy. An initial and continuing inflation-adjusted withdrawal of $10,000 per month plus a floating annual withdrawal of 2.4 percent of portfolio value (disbursed in equal monthly installments throughout the year). This results in an initial $20,000-per-month withdrawal level and a floating withdrawal level thereafter. The lower bound fixes the monthly withdrawal at $10,000; additional monthly withdrawals are a function of future portfolio value.

A front-loaded (glide path) withdrawal strategy. An initial annual inflation-adjusted withdrawal level of $360,000 (disbursed at a monthly rate of $30,000) decreasing annually each year by $12,000 until leveling off to a fixed $180,000 (with all values adjusted for inflation) in year 15 and thereafter.

Although conclusions remain both model-specific and case-specific, it appears that sequence of returns risk is operative over a variety of consumption and withdrawal strategies.

CONCLUSION

"Sequence" risk extends far beyond "sequence of returns" risk. It also encompasses risk factors listed earlier:

- Distribution and sequence of inflation realizations
- Distribution and sequence of retirement expenses
- Distribution of future annuity costs—sequence of yield curve changes
- Health state differential (bad to worse, bad to good, etc.)
- Force and effect of mortality.

Sequence risk, in its most expansive definition, is a fundamental reason for improving the financial services profession's portfolio supervision and client communication skill sets.[28]

The bear market case study depicts a race between principal depletion and the force of mortality. For modest-sized portfolios, an older client may experience more volatility in "sufficiency space" because the client consumes a greater percentage of principal whenever return expectations are not met. Furthermore, tracking portfolio feasibility in terms of the ACR means that identical portfolios can have very different risk characteristics. The annuity cost for a seventy-five-year-old may be significantly less expensive than for a sixty-five-year-old. Investment risk is age-related, as conventional wisdom suggests, but primarily as a function of the ACR value, not as a byproduct of a rule of thumb such as "equity weighting should be approximately 100 minus current age."

Sequence of returns risk evaluation is a complicated topic and it is difficult to identify the age range for which sequence order risk matters most. For example, for a forty-year-old consuming a small fraction of the portfolio (or not consuming at all) there is plenty of time to make up for a negative year. The low consumption won't take a big bite out of financial assets. Likewise, for an older investor, a bad upfront yearly return may not be a big deal because there are very few years left anyway, and consumption demands dominate capital gain/loss effects. However, at older ages mortality variability risk becomes very large,[29] and the investor potentially may be sensitive to a single bad investment year because both the consumption withdrawal and loss of principal put the portfolio on a catastrophic downward trajectory. The extent to which sequence of returns risk is operative is a facts-and-circumstances determination. Therefore, the value of the ACR as a monitoring benchmark. Sensitivity to this risk depends on the portfolio's distance from the feasibility boundary: ACR = 1.

Our retirement income risk modeling often reveals warning or danger zone areas that are high at the beginning of retirement, dip in the middle, and rise again at the end. Although it is impossible to articulate a universal rule, there is often greater likelihood of a negative outcome at the beginning and the end of retirement. The fact that feasible retirement spending does not move in lockstep with changes in wealth is due to factors other than sensitivity to capital market fluctuations. These include the following:

- Age, which changes in only one direction—no Benjamin Buttons are included in our retiree population. All else being equal, a ninety-year-old investor requires less wealth to produce a $20,000 monthly, inflation-adjusted lifetime income than does a seventy-year old investor.
- Changes in interest rates dynamically change the ACR value according to the interest-rate sensitivity of future liabilities and the interest-rate sensitivity of financial assets.

It is difficult, and perhaps dangerous, to conclude that avoiding a sequence of negative returns at the start of retirement means that the investor is home free. Whenever there is a possibility of significant deterioration in the ACR, a bear market can prove financially devastating. Stated otherwise, sequence risk for modest-sized portfolios often is present throughout retirement—especially at the beginning and the end.

Ultimately, of course, no investor is inherently interested in the evolution of ACR values over time; rather, they wish to know if they will be able to pay the bills in retirement. This argues for a behavioral finance framework for reporting future progress toward or regression from personal financial objectives. However, such a behaviorist communications framework must have analytical credibility, hence the need for a quantitative underpinning for feasibility and income sustainability such as outlined in this study. Part and parcel of this behaviorist orientation is to communicate risk metrics that actually reflect investor circumstances and concerns. We have neither presented nor discussed the lists of risk metrics found in many commercially available modeling packages. Appropriate risk metrics arise from each investor's financial goals and, to a large extent, are uniquely reflective of each investor's asset management elections. For example, investors lacking a strong gifting or bequest motivation usually are focused on income levels, income timing (e.g., more income early in retirement), income sustainability, income variation, income purchasing power, and so forth. For such income-oriented investors, utilizing investment and actuarial tools to monitor sequence of returns risk, to evaluate their vulnerabilities, and to assist them to implement prudent responses should suffice. Investors manifesting stronger gifting and bequest motivation challenge advisors to devise risk metrics that communicate progress toward or slippage away from these personal financial objectives.

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APPENDIX A: THE SIMULATION PROCESS AND RISK MODEL ASSUMPTIONS

Simulation is an approach to modeling that seeks to mimic a functioning system as it evolves. It is built on mathematical equations that express the assumed form of the system's operation. Simulation models assume a range of complexity from (1) a bootstrap of time-series data in which periodic returns are sampled with replacement to create a large number of reshuffled return sequences to (2) a structural model such as Monte Carlo simulation, which draws random samples from a pre-specified distribution to (3) more-complex simulation models that blend various types of distributions, or that switch among alternate distributions according to certain probability criteria. Whenever portfolios operate under conditions of cash flows (dollars going into or out of the portfolio), simulation analysis is an indispensable tool for evaluating the likelihood of economic success or failure.

The simulation model used in this paper incorporates several elements:

THE PLANNING HORIZON

The applicable planning horizon can be either fixed or variable. When the planning horizon is measured by life span, the model simulates sample lifetimes using a Society of Actuaries 2014 annuity table based on "white collar" retirees from defined benefit pension plans. This table is conservative (i.e., exhibits a force of mortality lower than general population tables used by Social Security), and therefore suggests a higher likelihood of a long life. Unless otherwise indicated, the simulation reflects longevity expectations that assume good health.

THE ECONOMY

The risk model divides economies into two regimes: A bear market regime (defined as a 20 percent or greater peak to trough price decline for the "Capital Appreciation S&P 500" stock index), and a bull market regime. Using historical data from January 1973 through 2016, the historical lengths of bull and bear markets are determined. The simulation uses a Markov-switching regime model that offers the user a choice between a market-agnostic perspective (with a random selection for the initial economic regime) and a bear market perspective (with an initial bear market economic regime). The probability (p) that the initial economy is in a bull market regime or a bear market regime ($1 - p$) is based on historical frequencies. For all future periods, the simulation determines the probability of remaining in a bear market given that the last month was a bear, or, of switching from a bear to a bull market given the total duration of the bear market to date. Similar calculations are made for the probability of remaining in or leaving a bull market regime.

INFLATION

The risk model approximates inflation by the Consumer Price Index (CPI). The econometric model specifies the inflation-generating process as a serially correlated random variable with a "smoothed" reversionary factor. Specifically, the algorithm regresses the average value of the previous twelve months' inflation against the average value of the next twelve months. The value is calculated as:

$$\text{Inflation} = \text{long-term inflation} + \text{Persistence Coefficient} \times [\text{Sum}(\text{inflation}_{t-1} \ldots \text{inflation}_{t-12}) / 12 - \text{long-term inflation}] + \text{error term}$$

where the error term is an independent and identically distributed (iid), "white noise" process.

When the application has not yet produced twelve monthly simulated values, the application recursively calculates the average of the preceding twelve months by using the initial value to replace any missing terms. Therefore, the value for average prior twelve-month inflation in the second month is 11/12 × the initial value + 1/12 × the value in the first month. The persistence coefficient determines the speed of CPI mean reversion. The coefficient's value is calculated via a regression of the rolling twelve-month CPI against the rolling forward twelve-month CPI. Thus the model assumes that inflation is a stochastic process that includes a term for autocorrelation as well as for mean reversion.

INVESTMENT RETURNS

The simulation model generates investment returns utilizing common matrix algebra techniques. Utilizing separate variance/covariance matrixes from historical bull and bear market regimes, the model executes, for each matrix, a Cholesky decomposition.[30] It also may adjust dependence relationships by shrinking extreme off-diagonal elements to assure matrix invertibility. The Cholesky matrix algebra operation "divides"

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a variance/covariance matrix into upper and lower triangle matrixes, which makes them equivalent to the square root of a variance matrix. If there exists a lower triangle matrix C such that the historical matrix $V = CC^t$, then C is a Cholesky matrix. The application simulates combinations of return series where each historical return series $(\vec{x})$ is transformed (by subtracting the mean and dividing by the standard deviation) into an independent standard normal variable $(\vec{z})$. The computer's random number generation function simulates future evolutions for each independent return vector by drawing values for uncorrelated zero-mean variables. Pre-multiplying the vectors of simulated independent returns by $C^tC(\vec{x})$ restores their equivalency to each original return series $(\vec{x}) = C(\vec{z})$. The variance of the independent vectors is easily determined; and, pre and post multiplication of the variance of $(\vec{z})$ by the appropriate lower triangle decomposition matrix C and its inverse restores the correlation structure by generating the required variance/covariance matrix. [$V = CV(\vec{z})C^t$].

The distributions, according to the Markov transition probabilities described above, enable the model to capture the risk of outlier results that mirror real world frequencies.

Financial asset return series usually cannot be characterized as normal (bell-curve) distributions. Portfolio investment risk defined by the first two moments of multivariate symmetric distributions (G aussian, Student's t, etc.) is often misleading. Monte Carlo simulations based on a normal distribution cannot realistically capture the frequency and magnitude of tail-risk events (leptokurtosis). To mitigate this deficiency, the application utilizes two normal distributions (bull and bear) with separately calculated means and variances for each regime. The distributions, according to the Markov transition probabilities described above, enable the model to capture the risk of outlier results that mirror real world frequencies.

Additionally, it is important to note that a regime-switching approach captures dynamic correlation and time-varying risk premia over different market conditions. Thus, instead of using average unconditional correlation values determined by the historical data, the risk model applies the historical correlation values conditioned on bull and bear market data. For example, over the entire sample period, an asset class may exhibit a mean of 10 percent and a standard deviation of 20 percent. However, during bull markets, the parameter values may be +18 percent mean and 15-percent standard deviation, while during bear markets, the parameter values may be –23 percent and 25 percent, respectively. Thus, simply using the unconditional mean, standard deviation, and correlation values for the aggregate historical period does not capture realistic asset price behavior.

FEES, EXPENSES, AND TRANSACTION COSTS

Advisor fees are deducted at a rate of 1 percent across all portfolio values. Transaction costs assume each asset class incurs trading costs and custodial fees of 1 basis point per month. Asset classes are implemented by pooled investment products such as exchange-traded funds and mutual funds. Annual expense ratios are: 84 basis points for U.S. equity, 112 basis points for foreign equity, and 36 basis points for fixed income.[31]

Finally, there is a factor for "implementation shortfall." Simplistically, implementation shortfall estimates the impact of bid-ask spreads and slippage incurred by trading in various capital markets. The risk model assumes a 40-basis-point shortfall for the U.S. large-cap equity market, a 60-basis-point shortfall for other U.S. equity markets, an 80-basis-point shortfall for international equity markets, and a 30-basis-point shortfall for fixed income capital markets.[32] Although the risk model can include the effects of taxes, in this paper we do not consider tax implications either from realized gain or loss, or from receipt of ordinary income dividends and interest. Nor do we consider the tax implications of locating assets in taxable or tax-favored investment accounts.

ANNUITIES

Annuity prices are an observable, market-based proxy for the cost of providing a lifetime income. The model benchmarks the cost of lifetime income by adding a 15-percent load factor to the estimated actuarial fair price of the specified lifetime income.

Although the model assumes that annuities are single premium, immediate contracts, it prices patterns of structured cash flows that commercially available contracts do not currently replicate (e.g., front-loaded and floating spending strategies). In these cases, the risk model uses the annuity pricing principle to estimate the cost of customized lifetime portfolio withdrawal patterns. The model specifies either a nominal or inflation-adjusted payout to the annuitant. It assumes that the owner and beneficiary are the same, and that contracts are owned by "natural persons" as defined in the U.S. Internal Revenue Code. Inflation-adjusted annuities increase benefits by a fixed 2.5 percent annual rate. Appendixes C and D provide additional information.

The simulation model inputs the above-listed elements to produce portfolio values over a wide range of possible future economies. Given the large number of simulation paths (10,000 trials), there is a rich set of future asset returns.

APPENDIX B: PORTFOLIO COMPOSITION

Table B1 ASSET ALLOCATION WEIGHTS FOR CASE STUDY 2

Asset Class	50% stock / 50% bond	Proxy Return Series
U.S. Large-Cap Stocks	8%	S&P 500
U.S. Large-Cap Value Stocks	8%	Fama French Large Value
U.S. Small-Cap Stocks	4%	CRSP 6-10
U.S. Small-Cap Value Stocks	4%	Fama French Small Value
U.S. Securitized Real Estate	4%	FTSE NAREIT All Equity REITs
International Large-Cap Stocks	8%	MSCI EAFE
International Large-Cap Value Stocks	4%	MSCI EAFE Value
International Small-Cap Stocks	4%	S&P EPAC Small
International Emerging Markets Stocks	4%	S&P/IFCI Composite
US Short-term Government T-Bills	19%	US 1 Year Treasury
US Intermediate-term Gov't Bonds	19%	BBgBarc US Gov't/Credit Interm.
Global Government Bonds	12%	FTSE World Gov't Bond Index

APPENDIX C: THE ANNUITY BENCHMARK

Annuity cost benchmarks present several challenges for retirement income risk modeling. A useful model is forward looking in that it provides insight into both the expected values of critical variables and the extent to which actual results may differ from expectations. Just as there is a probability distribution of investment returns, inflation rates, and investor life span, so, also, is there a distribution of future annuity costs.[33] Annuity prices are neither constant nor fully predictable from month-to-month, therefore, the cost of discharging the liability to provide future income is also a random variable.

Annuity prices are a function of several factors including the interest rate at the time of purchase, longevity expectations for the annuitant population, and the explicit and implicit costs of the annuity contract. The risk model must estimate current costs as well as project expected future costs. An expectation is a future-oriented value—it is a value that is more likely than not to be close to what is realized; a current cost estimate is a value that reflects the actual economic and demographic conditions of the day. In terms of model risk, the expected future annuity cost projection is a reasonable guideline for making long-term asset management decisions; the actual future price of an annuity is a measure of variance (risk) in the cost projection. *A priori*, we expect an increase in the magnitude of the difference between actual and expected during periods of abnormally low or high interest rates.

In one sense, finding the current price of an annuity is easy. Several websites allow the user to enter the requisite amount of monthly income as well as other contract features, including graded payment increases, in order to determine the contract's cost. Additionally, major custodians (e.g., Fidelity and TD Ameritrade) as well as mutual fund companies (e.g., Vanguard) provide prices for a select range of annuity contracts. The current annuity price determines the ratio of portfolio value to annuity cost—the ACR. Thus, if an investor currently owns a $2-million portfolio and obtains a $1-million market price for an immediate annuity providing the lifetime target income, the ACR value is 2 ÷ 1 = 2, or, a 2x coverage ratio. Any ratio value less than 1 signifies that current wealth is insufficient to fund the actuarial equivalent of the investor's target income stream. That is to say, from an actuarial perspective, the target income is not feasible given current resources, current interest rates, and other pricing factors including current age.[34]

However, an effective portfolio monitoring program must do more than simply present a single point-in-time annuity cost. As stated, the annuity cost is, itself, a stochastic variable. A monitoring and risk-evaluation program should estimate, over time, the ACR to provide insight into how the investor's preferred "safety margin" may unfold dynamically. For example, if the investor is committed to converting all or part of portfolio wealth to an annuity (or a bond ladder) if the ACR drops below 1.10, then examining the projected change in ratio value over time indicates the likelihood of a future conversion from financial assets to actuarial assets. Should annuitization not prove palatable, insight into the ratio value's trend provides useful information for determining if changes to asset allocation, investment or spending strategies, tax and asset location elections, and so forth are prudent. The ACR is a solvency benchmark, it is not a recommendation to buy an annuity contract.

APPENDIX D: PRICING ANNUITIES IN THE RETIREMENT INCOME RISK MODEL

Pricing annuities in a risk-modelling system presents significant challenges. The risk model described in this paper uses a pro forma yield curve based on four parameters: the current level of inflation, the real short-term rate premium to inflation, the real long-term rate premium, and the curvature from short- to long-term rates. A fundamental assumption is that both of these premiums (short term to inflation, and long term to short term) remain constant over time. The pro forma yield curve is not the current term structure of interest rates. Rather it produces an "expectation"—based on current inflation and historical risk premia—of what an investor might reasonably expect to pay, on average, for an annuity contract.

In the case of annuities, this assumption actually may be superior to assuming time-varying premiums reflecting forecasted changes in the yield curve's interest rates and shape. The purpose of the ACR calculation is to illustrate anticipated coverage ratios of portfolio assets to annuity costs. Introducing additional variance to the pro forma yield curve model would allow for a range of possible premiums which, in turn, would create noise around the expected annuity cost, and therefore noise around the expected portfolio/ACR. Therefore, the absence of variance is likely to provide a clearer answer to such questions as, "Will the investor be able to maintain a portfolio above the minimum asset level required to buy a replacement annuity?" without biasing the answer either positively or negatively.

The rate model used to price annuities is independent from the returns calculated for fixed income instruments. Both equity and fixed income returns are modeled on one-month normally distributed (and appropriately correlated) returns within the appropriate economic regime—bull or bear. The annuities are priced on a pro forma, non-varying forward yield curve model. As a result, the change in the price of an annuity is based on changes to the annuity buyer's age and on the current rate of inflation. During periods of high inflation, annuities are relatively cheaper because future payments are discounted at a higher rate. During periods of low inflation the converse is true. Of course, a client-specific monitoring system should periodically input updates to age and interest rates to recalculate annuity costs and, by extension, current and anticipated coverage ratios.

Modeling the evolution of the wealth-to-annuity-cost ratio requires several steps. The first step is to determine the "Starting Benefit Payment/Starting Price." The "Starting Benefit Payment" component equals the starting monthly benefit amount for a single premium immediate annuity purchased by the investor where the payment is measured in today's dollars. The starting periodic annuity payment may occur at month one (if the investor immediately opts to exchange financial wealth for annuity income) or, if the investor decides to defer the annuity purchase, the payment may start at any month thereafter. The "Starting Price" component automatically rolls forward month-by-month to project the annuity's cost as the investor ages, keeping current health status constant. Despite the nascent market for purchasing annuity and structured settlement income streams by private investment groups, we deem a decision to purchase an annuity to be an irrevocable wealth transfer.

The second modeling step is to "Adjust Starting Benefit Payment."[35] If an annuity purchase decision is deferred, the investor is faced with the need to increase the future-date initial periodic payment to offset a decrease in purchasing power. We adjust the future-date initial payment by the realized path of inflation as calculated by the retirement income risk model. For example, an investor could elect to purchase the annuity today at a wealth/annuity income ratio >1, or wait and see how the inflation-adjusted, wealth-adjusted ACR plays out in the future. That is to say, the coverage ratio is dynamic. An investor can estimate the likelihood that the ratio might either improve or deteriorate over time given the current asset allocation, target withdrawal amount, and projected mortality credits embedded within the annuity pricing structure. If the "Adjust Starting Benefit Payment" input is zero, then the model assumes that the annuity benefit is nominal rather than inflation adjusted. In a nutshell, the "Adjust Starting Benefit Payment" calculation reflects the amount of the annuity payment to be purchased. This is a key input into calculating how much wealth must be exchanged—now or in the future—for lifetime income. It is the information that the investor needs to consider in order to decide if and when annuitization is appropriate.

The final modeling step specifies the "Growth after Start" annuity benefit calculations. This is the change in annuity benefit after the contract payout start date. Because the risk model treats future inflation as a stochastic variable, the model requires a reasonable factor for increasing an annuity's future payouts once the annuity payout starts. If the input value is zero, the annuity payments, once started, do not increase. The risk model, in essence, asks the investor to elect an automatic yearly payout adjustment feature equal to 0 percent, 2 percent, 3 percent, 4 percent, or some other reasonable increase factor.[36] The larger the increase factor, the greater the annuity cost, all else being equal. Although the specified rate of payment increase is unlikely to match precisely the realized inflation rate, it nevertheless models costs for mitigating adverse effects on future purchasing power.

Prior to periodic update reviews with clients, the risk model allows the advisor to input the actual current yield curve data, current investor age, annuity type (nominal or graded benefit), and other information in order to update the current cost of providing lifetime income. Thus, the application makes an important distinction between the pro forma yield curve used for ACR projections and the actual yield curve data required for periodic portfolio monitoring and review.

APPENDIX E: CASE STUDY 2—ALTERNATE PORTFOLIO WITHDRAWAL STRATEGIES

Table E1 HYBRID WITHDRAWAL ELECTION HEAT MAP TABLE

	Years From Today										
	0.0Y	2.8Y	5.8Y	8.7Y	11.6Y	14.5Y	17.4Y	20.3Y	23.2Y	26.2Y	29.1Y
95%	150%	217%	253%	302%	371%	462%	581%	729%	908%	1156%	1348%
90%	150%	204%	237%	281%	344%	424%	532%	665%	832%	1031%	1280%
80%	150%	186%	216%	253%	307%	376%	467%	580%	717%	897%	1112%
70%	150%	174%	199%	232%	281%	341%	414%	505%	624%	758%	898%
60%	150%	163%	184%	214%	258%	307%	366%	445%	541%	635%	730%
50%	150%	152%	171%	197%	233%	275%	326%	384%	448%	503%	548%
40%	150%	141%	157%	179%	208%	243%	279%	317%	357%	365%	366%
30%	150%	129%	141%	159%	182%	207%	228%	240%	252%	222%	149%
20%	150%	114%	123%	135%	152%	160%	168%	163%	139%	82%	0%
10%	150%	94%	98%	104%	107%	104%	88%	39%	0%	0%	0%
5%	150%	73%	78%	78%	72%	61%	21%	0%	0%	0%	0%

Green = surplus; Yellow = awareness; Orange = caution; Red = danger
Alternative Strategy One: Hybrid Withdrawal Plan: $10,000 per month adjusted for inflation plus 2.4 percent of annual portfolio value paid monthly (initial year's withdrawal equals $20,000 per month)

Table E2 FRONT-LOADED (GLIDE PATH) WITHDRAWAL ELECTION HEAT MAP TABLE

	Years From Today										
	0.0Y	2.8Y	5.8Y	8.7Y	11.6Y	14.5Y	17.4Y	20.3Y	23.2Y	26.2Y	29.1Y
95%	126%	227%	304%	443%	712%	1231%	1152%	1638%	2189%	3068%	4164%
90%	126%	203%	267%	377%	591%	1018%	914%	1202%	1689%	2256%	3468%
80%	126%	176%	224%	312%	465%	779%	655%	818%	1063%	1383%	1798%
70%	126%	158%	198%	269%	386%	623%	485%	589%	704%	811%	909%
60%	126%	144%	177%	233%	324%	506%	363%	430%	481%	551%	619%
50%	126%	130%	158%	201%	270%	405%	274%	295%	304%	291%	369%
40%	126%	118%	139%	169%	216%	312%	196%	185%	154%	94%	27%
30%	126%	106%	120%	139%	168%	221%	117%	74%	9%	0%	0%
20%	126%	92%	98%	105%	113%	119%	33%	0%	0%	0%	0%
10%	126%	74%	70%	62%	43%	0%	0%	0%	0%	0%	0%
5%	126%	59%	52%	35%	0%	0%	0%	0%	0%	0%	0%

Green = surplus; Yellow = awareness; Orange = caution; Red = danger
Alternative Strategy Two: Front-Loaded (Glide Path) Withdrawal Plan: $30,000 per month ($360,000 initial year's withdrawal) adjusted for inflation decreasing $12,000 per year to $15,000 per month constant dollar withdrawal in year 15 and thereafter.

Each withdrawal strategy exhibits a "danger smile" pattern similar to that found in Case Study 2. Although a retired investor's portfolio withdrawal election remains an important asset management control element, most studies of retirement risk limit the option to a take more/take less choice. In a dynamic context within an asset-liability management framework, the scope of applicable options expands. Investors may have more flexibility (security) and, hence, retirement risk may decrease.

ENDNOTES

1. This paper is, in large part, based on material from the forthcoming book: *Retirement as a Risky Project: Monitoring and Managing a Retirement Income Portfolio*.
2. Thomas Bayes (1701–1761) was an English statistician, philosopher, and Presbyterian minister who is known for formulating a specific case of the theorem that bears his name: Bayes' theorem. In probability theory and statistics, Bayes' theorem (alternatively Bayes' law or Bayes' rule) describes the probability of an event, based on prior knowledge of conditions that might be related to the event. https://en.wikipedia.org/wiki/Thomas_Bayes.
3. Given what is known about the past, what is the investor's best guess about the future?

4. Of course, the investor's risk aversion also may change with changes in wealth and liabilities. This means that the investor, over time, updates preferences as well as beliefs.
5. In "input" or "response" terms, ACR provides a more complete view of the investor's economic circumstances. For example, a decrease in asset value caused by a change in the dividend discount rate may be mitigated, in part, by a decrease in liability values caused by higher interest rates. The ratio, rather than its individual components, is the key evaluation metric.
6. Target income can, of course, be a fixed periodic amount, a percentage-of-corpus ("unitrust") amount, a front-loaded or back-loaded glide path amount, or variations and combinations thereon, expressed in either nominal or inflation-adjusted dollars. Bottom line: Managing and monitoring a retirement income portfolio requires an initial financial analysis in terms of a client's balance sheet (risk capacity), an initial assessment of client financial goals and objectives (consumption/bequest preferences), initial probabilities for long-term sustainability of target threshold or aspirational standard-of-living cash flows (risk tolerance), and a rigorous monitoring and surveillance (goals-based evaluation of asset management options) and evaluation system. Over time, the center of attention shifts from strategic asset allocation policy toward feasibility/sustainability monitoring policy.
7. ACR value calculation can incorporate a number of deterministic or stochastic liabilities in addition to periodic spending objectives. See, for example, Yanikoski (2011) for a discussion about the incorporation of liabilities in a retirement income model and Bajtelsmit et al. (2013) in which the risk model incorporates a variety of expenses for health-related risk factors.
8. The use of an actuarial benchmark to gauge feasibility of cash-flow objectives has a long history in the literature. A detailed discussion of both investment-oriented probability measures of retirement income feasibility and sustainability (simulation), and actuarial-oriented measures of feasibility and sustainability (the annuity pricing principle) appears in Collins et al. (2015a). Both measures are critical because they often telegraph different signals to investors. In a low-interest-rate environment, when simulation engines show "green," actuarial calculations may show "red."
9. The ACR calculation parallels cost calculations for commercially available annuities paying either nominal or constant-dollar lifetime benefits. Establishing a credible ACR monitoring system for front- or back-loaded income streams, percentage of corpus portfolio ("unitrust") withdrawals, or combinations such as X dollars plus Y percent of portfolio value each year for life, requires sophisticated algorithms as discussed in appendixes C and D. Appendix E provides outputs using such algorithms within the context of Case Study 2.
10. The investor exhibits "temporal inconsistency" (Bordley and LiCalzi 2000). This observation is consistent with the Bayesian concepts discussed earlier.
11. Bordley and LiCalzi (2000, p. 63) point out that time horizon has an uncertain impact on option value. Increasing the time horizon often increases value, and uncertainty regarding the amount of the target itself (a stochastic target) may decrease option value. We believe that future research into the merits of flooring strategies should incorporate these issues. What is the price of a put option held by an individual where the "payoff" is defined as a financially secure retirement (i.e., the amount required to restore an underperforming portfolio to the feasibility level)? How should the option's value be calculated (e.g., the option's payoff is the ACR shortfall monetary calculation at a specified age)? How does the option's price compare with the price of a single-premium annuity contract, with a bond ladder, with a cash-flow matching strategy?
12. Longevity risk confronts any investor in danger of outliving limited resources irrespective of their age or expected longevity.
13. For example, $40,000 withdrawn at age seventy from a $1-million portfolio is a far smaller proportion of financial resources than $40,000 withdrawn at age eighty-five from a $300,000 portfolio.
14. Liabilities generally are defined as the need to provide periodic income. However, liabilities for late-in-life investors often increase because of health and disability issues.
15. See, also, the accompanying audience/participant discussion to Dempster and Medova (2011). "A male at 65 today has about a one-in-1000 chance of living to twice their life expectancy and life expectancy is, perhaps, 21 years. So at 65 are you going to live to 107? Probably not. But when you reach age 85, you have a one–in-ten chance of living to [twice] your life expectancy. In the same way that … we need to take account of the variability of returns on investment assets so, I believe, we may need to look at the variability of longevity …"
16. In this case, the initial annual return is the 2009 return while the last return in the sequence is the 1975 return. The return order is the reverse of the realized historical return order.
17. "Sustainable withdrawal rates are disproportionately explained by what happens in the early part of retirement. Returns from later in retirement have minimal impact …" (Pfau 2017, p. 6).
18. See the details in appendix A.
19. Technically, the survival distribution's probability mass exhibits greater right-side skew.
20. Although beyond the scope of this study, ACR values for investors in poor health can be estimated by increasing the investor's age for risk-modeling purposes. Age adjustment is common in the life insurance industry and it is also found in "underwritten annuities," also known as "substandard annuity contracts" or "impaired life annuities." These financial instruments are common in personal injury litigation settlements. See Ainslie (2000) and Brown and Scahill (2007).
21. This assumes comparable portfolios. In a "bucketing" approach to asset allocation, protecting against early downside risk by using low-volatility/low-return assets to fund initial consumption may generate a reverse glide path in which the portfolio's allocation to equity increases with investor age. An unintended result may arise if the older-age investor becomes more vulnerable to downside volatility as the planning horizon (remaining life expectancy) increases. A greater percentage of remaining wealth is exposed to equity risk as the planning horizon skews to the right.
22. An investor maintaining a fixed balance in a certificate of deposit finds that income fluctuates with each change in the interest rate offered at the certificate's renewal date. Over time, interest rates—retirement income—can vary enormously.
23. An extreme example is the transfer of all wealth to an insurance company for a fixed lifetime income guarantee. Spendable wealth disappears when an investor buys a contract to provide specified periodic future income. Exchanging wealth for a guaranteed lifetime income provides an example of a more-secure-with-less-wealth paradox.
24. Asset allocation weights are specified in appendix B.
25. We reiterate that ACR value calculation can incorporate a number of deterministic or stochastic liabilities in addition to periodic spending objectives. The book upon which we base this paper explores the feasibility and sustainability of a variety of spending strategies including percentage-of-portfolio spending, fixed + floating percentage combinations, front-loaded spending in early years of retirement, etc. See appendix E for further data on this case study.
26. See Friedman (2000). An interesting extension of free boundary problems involves the mathematics of black holes where the boundary location between normal space and the event horizon—the event horizon is the boundary at which light is no longer able to escape, so anything that penetrates the horizon is forever trapped—is both constantly shifting and unobservable. Passing through the event horizon is catastrophic. An interesting aspect of the event horizon is that you can pass through it without realizing it. Likewise, you could place yourself into what you believe to be a stable orbit only to spiral quickly into a black hole due to the smallest of perturbations. These nonlinear physical phenomena provide interesting perspectives into the portfolio risks discussed in this paper. Without an effective monitoring program, an investor can pass into the region of danger without realizing it or can believe that their retirement is on a stable economic footing just before a nerve-racking downward trajectory. For further insight into what happens when you encounter a black hole, see the University of Colorado website: http://jila.colorado.edu/~ajsh/insidebh/schw.html.
27. A random variable is a variable the future value of which you cannot predict with certainty. The possible future value is from a distribution of probabilities for possible values—a range over the space of possible outcomes as opposed to a single point estimate.
28. Sequence of returns risk, as traditionally defined, provides additional intellectual underpinning for portfolio-flooring and time-bucketing asset management strategies. A tilt toward fixed income early in retirement (the time during which the nest egg is, presumably, at its largest dollar value) undoubtedly has the effect of mitigating "left-tail" investment risk. At the start of retirement, selecting the asset allocation using this preferencing criterion allows the investor to achieve the optimal results with respect to the next time point (time 1). However, in the words of Christopher Rook (2014), such a

portfolio selection criterion is flawed because "… it acts optimally on a local basis and ignores the long-term impact of each short-term decision." During the initial retirement periods "… it would place the retiree into a low volatility portfolio attempting to minimize P(Ruin) before the next withdrawal. The inflation/expense-adjusted returns with such a portfolio will struggle to outperform the ruin factor, causing it to increase over time. Eventually, the ruin factor may spike and the strategy would respond by shifting the retiree into stocks reflecting desperation." Rook emphasizes that "Minimizing P(Ruin) by using sequential optimization fails because the [allocation] at time t=0 has an impact on the probability of ruin at later time points." In mathematical terms: "… a collection of local optimums does not necessarily aggregate to a global optimum." Rook outlines the interrelation between decision-making that seems optimal in the short term and decision-making that, in actuality, produces the most favorable long-term outcome. "Selecting a low volatility portfolio … early in retirement to avoid ruin at the next time point now comes with a price because that portfolio results in a higher value for the next ruin factor. … A larger ruin factor at the next time point increases the probability of ruin after the next time point." He recommends a dynamic programming approach to identify a balance point between the competing risk factors. Our paper introduces yet another element—the annuity pricing principle demonstrates how the option to delay annuitization, an extreme form of flooring—that becomes more valuable due to decreased annuity costs at older ages.

29. Ezra (2009) stresses this point. As retirees age they must set aside a bigger proportion of remaining wealth every year because the standard deviation of life expectancy is a bigger proportion of life expectancy at older ages.
30. A Cholesky decomposition factors a matrix of investment results into variance terms along its main diagonal and covariance terms in the off-diagonal. It is an invaluable tool for looking at the variance/co-variance of an entire portfolio rather than of investments in isolation.
31. Most passively managed mutual fund and exchange-traded fund expenses are lower. For an in-depth study of how changes in parameter values for fees, turnover rates, and taxes influence model output, see Collins et al. (2015b).
32. Edelen et al. (2013) estimate mean trading costs for U.S. equity mutual funds at 1.44 percent, where trading costs comprise commissions, spreads, and price impact. This is in addition to the average mutual fund expense ratio of 1.19 percent. Income tax implications from realized gain or loss from fund turnover is not considered in the analysis.
33. For a discussion of the distribution of future annuity cost, see Li (2008) and Koijen et al. (2011).
34. Simulations are investment projections, ACR values are actuarial projections. Both perspectives are helpful to arrive at prudent asset management strategies.
35. This is the label of an input field in the retirement income risk model.
36. As stated, we assume a 2.5-percent "growth after start" annual payment increase in this paper.

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